LECTURE 14 - INTRODUCTION TO FOUNDATIONS

14.1 OBJECTIVE OF LESSON

The purpose of this lesson is to introduce the design of spread footing and driven pile foundations required by the LRFD Specification. The lesson identifies general considerations which should be part of foundation design, outlines the design procedure, and presents requirements for the geotechnical and structural design of foundations at the service and strength limit states.

14.2 SPREAD FOOTING FOUNDATION DESIGN

14.2.1 General Design Considerations

In addition to the bearing resistance and load-settlement behavior of soil and rock that provide bearing support, the design of spread footing (or shallow) foundations requires consideration of local site constraints which can affect footing performance, including:

- Depth of the bottom of footing below ground surface to ensure adequate resistance to the effects of scour and frost;
- Anchorage of footings bearing on inclined rock surfaces by use of anchors, dowels or keys;
- Variable groundwater levels, including the effect of seepage when footings support walls which do not provide adequate drainage to prevent the build up of water behind the wall;
- Uplift loading; and
- The presence of nearby structures which are sufficiently close to the footing to impose load on the footing or could be influenced by the construction of or loading from the footing.

Of these factors, selection of the depth of embedment and evaluation of the variation of groundwater levels must be considered by the designer for most sites. If undermining by scour is a potential problem, footings must be constructed below the depth of scour. However, because accurate prediction of the depth of scour is difficult, the use of footings at these sites is permitted only if the footings are below scour depths determined for the check flood for scour per S2.6.4.4.2 and foundations supported on driven piles or drilled shafts should be considered. In regions of the U. S. where ground freezing is possible, foundations should be placed below the maximum depth of frost penetration to prevent damage.
from frost heave. Figure 14.2.1-1 shows the maximum depth of frost penetration in the continental U. S.

![Figure 14.2.1-1 - Maximum Depth of Frost Penetration](image)

Although most subsurface investigations include measurement of the groundwater level at the time of the exploration, the designer must recognize that the level varies with time and may not be at the maximum level at the time of the investigation. The periodic variation can be monitored using piezometers or estimated considering local variations in precipitation and the permeability of soils underlying the foundation. In regions of the U. S. where expansive or collapsible soils are present, variations in moisture content can result in substantial foundation movements if footings are constructed within the zone of influence of these moisture-sensitive soils.

The design of footings requires evaluation of footing movements at the Service Limit State and bearing resistance at the Strength Limit State. Because the design of most footings is controlled by settlement, consideration of settlement herein and in the Specification is presented first followed by evaluation of bearing resistance.

### 14.2.2 Design Procedure

The design procedure for spread footing foundations involves the following steps:

1. Develop design foundation profile including: soil and rock strata layering; engineering properties of foundation strata; groundwater level; and problem conditions.
2. Estimate loads for strength and service limit states analyses and select resistance factors for conditions analyzed.

3. Identify special conditions which need to be evaluated, such as loss of support due to scour or consolidation settlement of underlying cohesive soil layers.

4. Using presumptive bearing pressures or local experience, select preliminary footing width and length, and select footing depth based on location of suitable bearing strata and requirements to protect against freezing and/or scour of soils.

5. Estimate the settlement of the footing and compare with tolerable settlement. If the settlement is excessive, increase the footing width or revise the bearing level to achieve tolerable settlement.

6. Evaluate the bearing resistance of the foundation materials below the footing considering applicable load combinations.

7. Determine the location of the factored resultant force along the base of the footing and compare with eccentricity criterion. If the resultant load is beyond the acceptable limit, increase the footing width to meet eccentricity criterion.

8. Evaluate sliding resistance along base of footing due to inclined and lateral loads. If sliding resistance is not sufficient, increase footing width or improve footing contact by overexcavation and replacement of underlying materials to achieve acceptable sliding resistance.


Table 14.2.2-1 summarizes the strength and service limits states that must be evaluated for the design of spread footing foundations.
Table 14.2.2-1 - Strength and Service Limit States for Design of Spread Footing Foundations

<table>
<thead>
<tr>
<th>Design Consideration</th>
<th>Strength Limit State</th>
<th>Service Limit State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing Resistance</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Settlement</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Sliding Resistance</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Overturning</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>(Eccentricity of Base Pressure Resultant)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structural Capacity</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

14.2.3 Movement and Bearing Pressure at Service Limit State

14.2.3.1 ANALYSIS OF FOOTING MOVEMENTS

The movement of footings shall be evaluated at the service limit state using the Service I Load Combination from Table S3.4.1-1 in LRFD. Thus, the vertical settlement of footings should be estimated. The potential for lateral displacement of footings should also be considered where the:

- Footing is subjected to inclined or vertical loads;
- Footing is placed on or near an embankment slope;
- Loss of foundation support is possible by scour; and
- Bearing strata (usually rock) are significantly inclined.

Depending on the material providing foundation support, square or nearly square footings stress the foundation material to a depth of about two footing widths (i.e., 2B), and long footings (i.e., L/B > 5) stress the foundation material to a depth of about 4B.

The vertical settlement will be a combination of the elastic, consolidation and secondary compression movements. In general, settlement of footings on cohesionless soils, very stiff to hard cohesive soils, and rock with tight, unfilled joints will be elastic and will occur as load is applied. For footings on very soft to stiff cohesive soils, the potential for consolidation and secondary compression settlement components should be evaluated in addition to elastic settlement.
Methods for estimating settlement of footings are described in Barker, et al, (1991), as well as various geotechnical engineering textbooks (e.g., Hunt, 1986; Holtz and Kovacs, 1981; and Fang, 1991). The elastic settlements of footings are estimated using elastic theory and a value of elastic modulus based on the results of in-situ or laboratory testing. The consolidation and secondary compression settlement of footings on cohesive soils is estimated using consolidation theory and the results of laboratory consolidation tests.

As an aid in estimating approximate footing dimensions for settlement analysis, Table 14.2.3.1-1 presents presumptive bearing pressures for various types of foundation material. After the approximate footing dimensions are determined, settlement of the footing can be determined as described in S10.6.2.2.3 in LRFD for footings on cohesionless and cohesive soils and rock.

Table 14.2.3.1-1 - Presumptive Bearing Pressures for Spread Footings at the Service Limit State

<table>
<thead>
<tr>
<th>TYPE OF BEARING MATERIAL</th>
<th>CONSISTENCY IN PLACE</th>
<th>BEARING PRESSURE (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ordinary Range</td>
<td>Recommended Value of Use</td>
</tr>
<tr>
<td>Massive crystalline igneous and metamorphic rock: graphite, diorite, basalt, gneiss, thoroughly cemented conglomerate (sound condition allows minor cracks)</td>
<td>Very hard, sound rock</td>
<td>5.7 to 9.6</td>
</tr>
<tr>
<td>Foliated metamorphic rock: slate, schist (sound condition allows minor cracks)</td>
<td>Hard sound rock</td>
<td>2.9 to 3.8</td>
</tr>
<tr>
<td>Sedimentary rock: hard cemented shales, siltstone, sandstone, limestone without cavities</td>
<td>Hard sound rock</td>
<td>1.4 to 2.4</td>
</tr>
<tr>
<td>Weathered or broken bedrock of any kind, except highly argillaceous rock (shale)</td>
<td>Medium hard rock</td>
<td>0.77 to 1.1</td>
</tr>
<tr>
<td>TYPE OF BEARING MATERIAL</td>
<td>CONSISTENCY IN PLACE</td>
<td>BEARING PRESSURE (MPa)</td>
</tr>
<tr>
<td>--------------------------</td>
<td>----------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ordinary Range</td>
</tr>
<tr>
<td>Compaction shale or other highly argillaceous rock in sound condition</td>
<td>Medium hard rock</td>
<td>0.77 to 1.1</td>
</tr>
<tr>
<td>Well-graded mixture of fine- and coarse-grained soil: glacial till, hardpan, boulder clay (GW-GC, GC, SC)</td>
<td>Very dense</td>
<td>0.77 to 1.1</td>
</tr>
<tr>
<td>Gravel, gravel-sand mixture, boulder-gravel mixtures (GW, GP, SW, SP)</td>
<td>Very dense</td>
<td>0.57 to 0.96</td>
</tr>
<tr>
<td>Coarse to medium sand, and with little gravel (SW, SP)</td>
<td>Very dense</td>
<td>0.38 to 0.57</td>
</tr>
<tr>
<td>Fine to medium sand, silty or clayey medium to coarse sand (SW, SM, SC)</td>
<td>Very dense</td>
<td>0.29 to 0.48</td>
</tr>
<tr>
<td>Fine sand, silty or clayey medium to fine sand (SP, SM, SC)</td>
<td>Very dense</td>
<td>0.29 to 0.48</td>
</tr>
<tr>
<td>Homogeneous inorganic clay, sandy or silty clay (CL, CH)</td>
<td>Very stiff to hard</td>
<td>0.29 to 0.57</td>
</tr>
<tr>
<td>Inorganic silt, sandy or clayey silt, varied silt-clay-fine sand (ML, MH)</td>
<td>Very stiff to hard</td>
<td>0.19 to 0.38</td>
</tr>
</tbody>
</table>

14.2.3.2 MOVEMENT CRITERIA

The tolerable movement of shallow foundations depends on structural criteria, such as the type and size of the supported superstructure, as well as factors, such as the cost and difficulty of implementing repairs in the future, rideability, aesthetics and safety. Based on such factors, limits on foundation movements have been set arbitrarily or based on empirical assumptions (e.g., 25 mm maximum vertical movement) without consideration of actual
structural performance. However, due to the effects of creep, relaxation and redistribution of forces in bridge superstructures, bridges can accommodate substantially more settlement than traditionally allowed or anticipated in design. This conclusion is supported by the results of FHWA research (Moulton, et al, 1985) which involved a performance survey of more than 200 bridges supported on shallow foundations. The study showed that angular distortions (i.e., relative settlement of adjacent foundations divided by the span length) of 0.008, or less for simple span structures and 0.004 or less for continuous-span structures, are acceptable. Using these relationships, the maximum tolerable settlement between foundations can be much greater than normally assumed. The final choice for permissible movement should be made by the bridge designer in consultation with a geotechnical engineer.

14.2.4 Bearing and Sliding Resistance at the Strength Limit State

Bearing resistance pertains to the capacity of the foundation materials to support loads distributed from the structure and loads from other sources. If the foundation materials supporting a footing provide adequate bearing resistance, the design will ensure that bearing failure (and associated unacceptable movements) should not occur due to shear distortion in the ground below the footing.

Foundations must be designed to safely support the loads they carry. Accordingly, the foundation must not fail structurally and must provide adequate bearing support for all applicable loading conditions. Structural failure can be avoided by assuring the foundation has sufficient shear and moment capacity to distribute the load into the foundation materials. Evaluation of the structural resistance of foundations is presented in Sections 5, 6 and 8 of the LRFD Specification.

14.2.4.1 RESISTANCE FACTORS

Resistance factors for the geotechnical design of shallow foundations are presented in Table 14.2.4.1-1. A majority of the resistance factors in the table are based on design procedures which are commonly used for allowable stress design (ASD) of shallow foundations. The resistance factors presented in Table 14.2.4.1-1 were developed mostly by calibration with reliability theory where sufficient statistical information was available regarding particular design procedures, tempered with engineering judgment for some cases. Where statistical information was insufficient, resistance factors were chosen by calibration with working stress design so that LRFD and current design would result in a footing with similar dimensions.
14.2.4.2 BEARING RESISTANCE

The LRFD Specification requires that the bearing resistance of a footing subjected to vertical loads be evaluated at the strength limit state using the appropriate strength and/or extreme event load combinations from Table S3.4.1-1 and resistance factors from Table 14.2.4.2-1. If a footing is subjected to horizontal or inclined loading, the design must also provide adequate resistance to sliding (see 14.2.4.4).
<table>
<thead>
<tr>
<th>METHOD/SOIL/CONDITION</th>
<th>RESISTANCE FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bearing Capacity and Passive Pressure</strong></td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td></td>
</tr>
<tr>
<td>- Semi-empirical procedure using SPT data</td>
<td>0.45</td>
</tr>
<tr>
<td>- Semi-empirical procedure using CPT data</td>
<td>0.55</td>
</tr>
<tr>
<td>- Rational Method --</td>
<td></td>
</tr>
<tr>
<td>using $\phi_f$ estimated from SPT data</td>
<td>0.35</td>
</tr>
<tr>
<td>using $\phi_f$ estimated from CPT data</td>
<td>0.45</td>
</tr>
<tr>
<td>Clay</td>
<td></td>
</tr>
<tr>
<td>- Semi-empirical procedure using CPT data</td>
<td>0.50</td>
</tr>
<tr>
<td>- Rational Method using shear resistance measured in lab tests</td>
<td>0.60</td>
</tr>
<tr>
<td>using shear resistance measured in field vane tests</td>
<td>0.60</td>
</tr>
<tr>
<td>using shear resistance estimated from CPT data</td>
<td>0.50</td>
</tr>
<tr>
<td>Rock</td>
<td></td>
</tr>
<tr>
<td>- Semi-empirical procedure, Carter and Kulhawy (1988)</td>
<td>0.60</td>
</tr>
<tr>
<td>Plate Load Test</td>
<td>0.55</td>
</tr>
<tr>
<td>Sliding</td>
<td></td>
</tr>
<tr>
<td>Precast concrete placed on sand:</td>
<td></td>
</tr>
<tr>
<td>using $\phi_f$ estimated from SPT data</td>
<td>0.90</td>
</tr>
<tr>
<td>using $\phi_f$ estimated from CPT data</td>
<td>0.90</td>
</tr>
<tr>
<td>Concrete cast-in-place on sand:</td>
<td></td>
</tr>
<tr>
<td>using $\phi_f$ estimated from SPT data</td>
<td>0.80</td>
</tr>
<tr>
<td>using $\phi_f$ estimated from CPT data</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Sliding on clay is controlled by the strength of the clay when the clay shear is less than 0.5 times the normal stress, and is controlled by the normal stress when the clay shear is greater than 0.5 times the normal stress (see Figure 1).

Clay (where shear resistance is less than 0.5 times normal pressure)

- using shear resistance measured in lab tests: 0.85
- using shear resistance measured in field tests: 0.85
- using shear resistance estimated from CPT data: 0.80

Clay (where the resistance is greater than 0.5 times normal pressure): 0.85

Passive earth pressure component of sliding resistance: 0.50

Overall Stability
Shallow foundations on or near a slope evaluated for overall stability and resistance to a deep-seated failure mode: 0.85

The nominal bearing resistance (or ultimate bearing capacity, q_{ult}) of the foundation materials is determined using existing procedures from ASD for estimating q_{ult}. The Specification provides guidance for the determination of q_{ult} based on theoretical estimation procedures using soil properties based on test results and on in-situ test procedures using the SPT, CPT and pressuremeter. The design procedures can include the effects of:

- footing shape
- load inclination
- depth of footing embedment
- soil compressibility
- position of the groundwater table below the footing
• load eccentricity

For footings supported on saturated or nearly saturated cohesive soils, the unfactored value of $q_{ult}$ can be determined as:

$$q_{ult} = cN_{cm} + g\gamma D_f N_{qm} \times 10^{-9} \quad (14.2.4.2-1)$$

where:

- $q_{ult}$ = ultimate or nominal bearing capacity (MPa)
- $c = S_u =$ Undrained shear strength (MPa)
- $N_{cm}, N_{qm}$ = modified bearing capacity factors which account for the effects of footing shape, embedment depth, soil compressibility and load inclination (dim)
- $g = \text{gravitational acceleration constant} \ (9.8066 \text{ m/sec}^2)$
- $\gamma =$ total density of cohesive soil (kg/m$^3$)
- $D_f =$ embedment depth to footing bottom (mm)

The bearing capacity factors $N_{cm}$ and $N_{qm}$ are defined as:

$$N_{cm} = 5[1 + 0.2(D_f/B)][1 + 0.2(B/L)][1 - 1.3(H/V)] \text{ for } D_f/B \leq 2.5, B/L \leq 1 \text{ and } H/V \leq 0.4; \quad (14.2.4.2-2)$$

$$N_{cm} = 7.5[1 + 0.2(B/L)][1 - 1.3(H/V)] \text{ for } D_f/B > 2.5 \text{ and } H/V > 0.4 \quad (14.2.4.2-3)$$

$$N_{qm} = 1.0 \text{ for saturated clay} \quad (14.2.4.2-4)$$

For footings supported on cohesionless soils, the unfactored value of $q_{ult}$ can be determined as:

$$q_{ult} = 0.5g\gamma B C_{w1} N_{ym} \times 10^{-9} + g\gamma C_{w2} D_f N_{qm} \times 10^{-9} \quad (14.2.4.2-5)$$

where:

- $B =$ width of footing (mm)
- $L =$ length of footing
- $C_{w1}, C_{w2} =$ coefficients for depth of groundwater level below footing (dim)
\[ N_{ym}, N_{qm} = \text{modified bearing capacity factors which account for the effects of footing width and footing embedment (dim)} \]

\[ \gamma = \text{total density of cohesionless soil (kg/m}^3\text{)} \]

The bearing capacity factors \( N_{ym} \) and \( N_{qm} \) are defined as:

\[ N_{ym} = N_{\gamma} s y c i d_{\gamma} \quad (14.2.4.2-6) \]

\[ N_{qm} = N_{qs} q c i d_{qs} \quad (14.2.4.2-7) \]

where values of each term can be interpolated from Tables 14.2.4.2-1 through 14.2.4.2-8.

Table 14.2.4.2-1 - Bearing Capacity Factors \( N_{\gamma} \) and \( N_{q} \) for Footings on Cohesionless Soil

(Barker, et al, 1991)

<table>
<thead>
<tr>
<th>Friction Angle, ((\varphi_{f})) (deg)</th>
<th>(N_{\gamma}) (dim)</th>
<th>(N_{q}) (dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td>30</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>32</td>
<td>30</td>
<td>23</td>
</tr>
<tr>
<td>34</td>
<td>41</td>
<td>29</td>
</tr>
<tr>
<td>36</td>
<td>58</td>
<td>38</td>
</tr>
<tr>
<td>38</td>
<td>78</td>
<td>49</td>
</tr>
<tr>
<td>40</td>
<td>110</td>
<td>64</td>
</tr>
<tr>
<td>42</td>
<td>155</td>
<td>85</td>
</tr>
<tr>
<td>44</td>
<td>225</td>
<td>115</td>
</tr>
<tr>
<td>46</td>
<td>330</td>
<td>180</td>
</tr>
</tbody>
</table>
Table 14.2.4.2-2 - Shape Factor $s_q$ for Footings on Cohesionless Soil

(Barker, et al, 1991)

<table>
<thead>
<tr>
<th>Friction Angle, ($\phi_f$) (deg)</th>
<th>$s_q$ (dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L/B = 1</td>
</tr>
<tr>
<td>28</td>
<td>1.53</td>
</tr>
<tr>
<td>30</td>
<td>1.58</td>
</tr>
<tr>
<td>32</td>
<td>1.62</td>
</tr>
<tr>
<td>34</td>
<td>1.67</td>
</tr>
<tr>
<td>36</td>
<td>1.73</td>
</tr>
<tr>
<td>38</td>
<td>1.78</td>
</tr>
<tr>
<td>40</td>
<td>1.84</td>
</tr>
<tr>
<td>42</td>
<td>1.90</td>
</tr>
<tr>
<td>44</td>
<td>1.96</td>
</tr>
<tr>
<td>46</td>
<td>2.03</td>
</tr>
</tbody>
</table>

Table 14.2.4.2-3 - Shape Factor $s_\gamma$ for Footings on Cohesionless Soil

(Barker, et al, 1991)

<table>
<thead>
<tr>
<th>L/B</th>
<th>$s_\gamma$ (dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.60</td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
</tr>
<tr>
<td>5</td>
<td>0.92</td>
</tr>
<tr>
<td>10</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Table 14.2.4.2-4 - Soil Compressibility Factors $c_y$ and $c_q$ for Square Footings on Cohesionless Soil

(Barker, et al, 1991)

<table>
<thead>
<tr>
<th>Relative Density (%)</th>
<th>Friction Angle, $(\phi_y)$ (deg)</th>
<th>$c_y = c_q$ (dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$q = 0.024$ MPa</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>1.00</td>
</tr>
<tr>
<td>30</td>
<td>32</td>
<td>1.00</td>
</tr>
<tr>
<td>40</td>
<td>35</td>
<td>1.00</td>
</tr>
<tr>
<td>50</td>
<td>37</td>
<td>1.00</td>
</tr>
<tr>
<td>60</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td>70</td>
<td>42</td>
<td>0.96</td>
</tr>
<tr>
<td>80</td>
<td>45</td>
<td>0.79</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Note: $q$ equals the initial vertical effective stress at the footing depth, i.e., vertical stress at bottom of footing prior to excavation, corrected for water pressure.

Table 14.2.4.2-5 - Soil Compressibility Factors $c_y$ and $c_q$ for Strip Footings on Cohesionless Soil

(Barker, et al, 1991)

<table>
<thead>
<tr>
<th>Relative Density (%)</th>
<th>Friction Angle, $(\phi_y)$ (deg)</th>
<th>$c_y = c_q$ (dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$q = 0.024$ MPa</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>0.85</td>
</tr>
<tr>
<td>30</td>
<td>32</td>
<td>0.80</td>
</tr>
<tr>
<td>40</td>
<td>35</td>
<td>0.76</td>
</tr>
<tr>
<td>50</td>
<td>37</td>
<td>0.73</td>
</tr>
<tr>
<td>60</td>
<td>40</td>
<td>0.62</td>
</tr>
<tr>
<td>70</td>
<td>42</td>
<td>0.56</td>
</tr>
<tr>
<td>80</td>
<td>45</td>
<td>0.44</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Note: $q$ equals the initial vertical effective stress at the footing depth.
Table 14.2.4.2-6 - Load Inclination Factors $i_y$ and $i_q$ for Load Inclined in Direction of Footing Width

(Barker, et al, 1991)

<table>
<thead>
<tr>
<th>H/V</th>
<th>$i_y$ (dim)</th>
<th>$i_q$ (dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strip</td>
<td>L/B = 2</td>
</tr>
<tr>
<td>0.10</td>
<td>0.73</td>
<td>0.76</td>
</tr>
<tr>
<td>0.15</td>
<td>0.61</td>
<td>0.65</td>
</tr>
<tr>
<td>0.20</td>
<td>0.51</td>
<td>0.55</td>
</tr>
<tr>
<td>0.25</td>
<td>0.42</td>
<td>0.46</td>
</tr>
<tr>
<td>0.30</td>
<td>0.34</td>
<td>0.39</td>
</tr>
<tr>
<td>0.35</td>
<td>0.27</td>
<td>0.32</td>
</tr>
<tr>
<td>0.40</td>
<td>0.22</td>
<td>0.26</td>
</tr>
<tr>
<td>0.45</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>0.50</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>0.55</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>0.60</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td>0.65</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>0.70</td>
<td>0.03</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Note: $H$ and $V$ are unfactored loads.

Table 14.2.4.2-7 - Load Inclination Factors $i_y$ and $i_q$ for Load Inclined in Direction of Footing Length

(Barker, et al, 1991)

<table>
<thead>
<tr>
<th>H/V</th>
<th>$i_y$ (dim)</th>
<th>$i_q$ (dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strip</td>
<td>L/B = 2</td>
</tr>
<tr>
<td>0.10</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td>0.15</td>
<td>0.72</td>
<td>0.68</td>
</tr>
<tr>
<td>0.20</td>
<td>0.64</td>
<td>0.59</td>
</tr>
<tr>
<td>0.25</td>
<td>0.56</td>
<td>0.51</td>
</tr>
<tr>
<td>0.30</td>
<td>0.49</td>
<td>0.44</td>
</tr>
<tr>
<td>0.35</td>
<td>0.42</td>
<td>0.37</td>
</tr>
<tr>
<td>0.40</td>
<td>0.36</td>
<td>0.30</td>
</tr>
<tr>
<td>0.45</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>0.55</td>
<td>0.20</td>
<td>0.16</td>
</tr>
<tr>
<td>0.60</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>0.65</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>0.70</td>
<td>0.09</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Note: $H$ and $V$ are unfactored loads.
Table 14.2.4.2-8 - Depth Factor \(d_q\) for Cohesionless Soils

(Barker, et al, 1991)

<table>
<thead>
<tr>
<th>Friction Angle, ((\phi_f))</th>
<th>D/B (dim)</th>
<th>(d_q) (dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>1</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.40</td>
</tr>
<tr>
<td>37</td>
<td>1</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.35</td>
</tr>
<tr>
<td>42</td>
<td>1</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Note: Values of \(d_q\) are applicable if the soils above the footing bottom are as competent as the soils below the footing. If the soils are weaker, use \(d_q = 1.0\).

The nominal bearing resistance, \(R_n\), of the foundation materials is determined by multiplying \(q_{ult}\) by the footing area. The adequacy of the footing at the strength limit state is then computed by multiplying \(R_n\) by the appropriate resistance factor, \(\phi\), and determining whether the factored resistance equals or exceeds the factored loads using the design equation for LRFD. If the factored resistance is less than the factored loads, the footing size must be increased until design equation for LRFD is satisfied.

14.2.4.3 LOAD ECCENTRICITY

If the location of load application is eccentric to the centroid of the footing, compliance of the footing design with tolerable movement criteria and adequate bearing resistance must be based on a reduced effective footing area (i.e., \(B' \times L'\)) within the dimensions of the footing so that the load is acting at the centroid of the reduced area. For the purposes of geotechnical design, the bearing pressure can be assumed to be uniformly applied to the effective footing area. As shown in Figure 14.2.4.3-1, the reduced dimensions for an eccentrically loaded footing can be taken as:

\[
B' = B - 2e_B
\]  

(14.2.4.3-1)
\[ L' = L - 2e_L \]

Figure 14.2.4.3-1 - Reduced Footing Dimensions

where \( e_B \) and \( e_L \) are the eccentricities parallel to the B and L dimensions, respectively. On this basis, footings subjected to eccentric loads must be designed to ensure that:

- The factored bearing resistance is not less than the factored loads; and
- Except for the extreme event limit states, the eccentricity of footings based on factored loads is less than \( \frac{B}{4} \) or \( \frac{L}{4} \) for footings on soil, and less than \( \frac{3B}{8} \) or \( \frac{3L}{8} \) for footings on rock. More liberal eccentricities may be allowed for the extreme event limit state.

For structural design, the bearing pressure at the base of the footing is usually assumed to vary linearly across the bottom of the
footing. This assumption results in a slightly conservative triangular or trapezoidal contact pressure distribution.

14.2.4.4 SLIDING RESISTANCE

If footings support inclined loads or are founded on an inclined base, failure by sliding along the footing base must be evaluated. As described in S10.6.3.3, the factored resistance against failure by sliding, $Q_R$, is determined as:

$$Q_R = (\varphi_r Q_r + \varphi_{ep} Q_{ep})$$  \hfill (14.2.4.4-1)

where:

- $\varphi_r$ = resistance factor for shear between the footing and foundation material from Table S10.5.4-1
- $Q_r$ = nominal shear resistance between footing and foundation material (N)
- $\varphi_{ep}$ = resistance factor for passive resistance from Table 14.2.4.1-1
- $Q_{ep}$ = nominal passive resistance of foundation material available throughout the design life of the footing (N)

For most cases, the passive resistance component of sliding resistance is ignored or reduced because it is difficult to assure that loss of ground (e.g., temporary excavation) or loss of contact by shrinkage will not occur in the future.

For footings supported on cohesionless soil, $Q_r$ can be determined as:

$$Q_r = V \tan\delta$$  \hfill (14.2.4.4-2)

where:

- $V$ = total vertical force (N)
- $\tan\delta$ = $\tan\varphi_f$ for concrete cast directly against soil, or $0.8\tan\varphi_f$ for a precast footing
- $\varphi_f$ = soil angle of friction (deg)

For footings supported on cohesive soil, $Q_r$ can be determined as the lesser of:

- the cohesion of the foundation soil, or
• one-half the normal stress on the interface between the footing and soil as shown in Figure 14.2.4.4-1 below.

\[ q_s = \text{unit shear resistance, equal to } S_u \text{ or } 0.5 \sigma_v', \text{ whichever is less} \]

\[ Q_r = \text{area under } q_s \text{ diagram (shaded area)} \]

\[ S_u = \text{undrained shear strength (MPa)} \]

\[ \sigma_v' = \text{vertical effective stress (MPa)} \]

Figure 14.2.4.4-1 - Procedure for Estimating Sliding Resistance of Footings on Clay

It should be noted that the latter case is applicable only when the footing is supported on a thin (e.g., 150 mm) thickness of compacted granular soil prior to footing construction.

Evaluation of footing failure by sliding is necessary, for example, for the design of a retaining wall. For this case, the magnitudes of earth load which tend to stabilize and destabilize the footing can be determined as described in Article S3.11.5 of the Specification and Section 14.2.2 herein.
14.3 DRIVEN PILE FOUNDATIONS DESIGN

14.3.1 General Design Considerations

In addition to determining the design pile section to meet the service and strength limit states criteria for axial and lateral loading, the design of driven pile foundations required consideration of factors which can affect pile performance including:

- pile spacing and the effects of group action (S10.7.15, S10.7.3.8 and S10.7.3.10)
- the possibility of scour and its effect on capacity and movement for piles installed near or into flowing water channels (S10.7.1.3)
- negative or downdrag loads resulting from piles driven through soft compressible soil which are settling due to applied surcharges (S10.7.1.4)
- the effect of variable groundwater levels and buoyancy (S10.7.1.7)
- protection against deterioration due to corrosion of steel piles, sulfate, chloride and acid attack on concrete piles, and decay of timber piles due to wet/dry cycles and insects (S10.7.1.8)
- uplift loading of single piles and pile groups (S10.7.1.9 and S10.7.3.7)
- battering of piles to improve the resistance of a pile group to lateral loading (S10.7.1.6 and S10.7.3.9)

14.3.2 Design Procedure

The design procedure for driven pile foundations involves the following steps:

1. Develop design foundation profile including soil and rock strata layering, engineering properties of foundation strata, groundwater level and problem conditions.

2. Estimate loads for strength and service limit states analyses and select resistance factors for conditions analyzed.

3. Identify special conditions which need to be evaluated, such as loss of lateral support due to scour or negative loading on end bearing piles due to settling ground.
4. Select pile type(s) and lengths suitable for site conditions, local availability and cost.

5. Estimate axial (compression) pile capacity considering both soil/rock and structural capacity, and investigate drivability.

6. Determine the number, location and spacing of piles in a group required to support factored load combinations.

7. Estimate pile group efficiency and adjust number and spacing of piles to provide adequate capacity.

8. For bearing pile group, check for possible punching of pile group into weak stratum underlying bearing level.

9. Estimate the settlement of the pile group and compare with tolerable settlement. If the settlement is excessive, revise the pile design by increasing the length and/or number of piles.

10. If pile group is subjected to uplift loading, check uplift capacity.

11. Evaluate the structural capacity of pile group to lateral loading.

12. Estimate the lateral displacement of the pile group and compare with tolerable displacement. If the displacement is excessive, revise the pile design by increasing the length and/or number of piles.

13. Perform structural design of pile in accordance with Sections 5, 6 or 8 of LRFD depending on pile type.

14. Determine whether special construction control is required, such as dynamic monitoring or static load testing.

Table 14.3.2-1 summarizes the strength and service limits states that must be evaluated for the design of driven pile foundations.
Table 14.3.2-1 - Strength and Service Limit States for Design of Driven Pile Foundations

<table>
<thead>
<tr>
<th>Design Consideration</th>
<th>Strength Limit State</th>
<th>Service Limit State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing Resistance of Single Pile/Group</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Structural Capacity of Single Pile</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Punching</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Settlement of Pile Group</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Tensile Capacity of Uplift-Loaded Piles</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Structural Capacity of Laterally-Loaded Piles</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Lateral Displacement of Pile or Pile Group</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

14.3.3  Movement at the Service Limit State

14.3.3.1  ANALYSIS OF PILE DISPLACEMENTS

The displacement of driven piles in cohesionless soil shall be evaluated for all applicable load combinations using the Service I Load Combination from Table S3.4.1-1. For piles in cohesive soil, the requirement to consider transient loads may be omitted. The axial displacement of piles should be estimated. The lateral displacement of piles should also be evaluated where the:

- pile is subjected to inclined or vertical loads,
- pile is placed on or near an embankment slope, and
- loss of lateral foundation support by scour is possible.

The Specification provides guidance for estimating the settlement of pile groups in cohesive and cohesionless soils. The procedure for estimating the settlement of a pile group in cohesive soil uses an equivalent footing for the pile group as shown in Figure 14.3.3.1-1. When the equivalent footing procedure is used, the methods presented in S10.6.2 for estimating the immediate and consolidation settlements of footings on cohesive soil should be followed.
For a pile group in cohesionless soil, semi-empirical procedures are presented in LRFD which are based on the results of SPT and CPT in-situ testing. If SPT results are available, the settlement of a pile group bearing in sand can be estimated using:

\[
D' = \frac{360 q I \sqrt{X}}{N_{corr}}
\]

(14.3.3.1-1)

where:
\[ I' = 0.125 \frac{D}{X} \# 0.5 \]  

(14.3.3.1-2)

\[ N_{corr} = 0.77 \log_{10} \left( \frac{1.92}{\sigma_v} \right) \]

(14.3.3.1-3)

where:

\[ \rho \] = settlement of pile group (mm)

\[ q \] = net foundation pressure applied at a depth of 2\( D_b \)/3 as shown in Figure 14.3.3.1-1 (MPa)

\[ X \] = width or smallest dimension of pile group (mm)

\[ I \] = influence factor of the effective group embedment (dim)

\[ D_N \] = effective depth taken as 2\( D_b \)/3 (mm)

\[ D_b \] = depth of pile group embedment (mm)

\[ N_{corr} \] = average corrected SPT blow count within a depth of \( X \) below the group (blows/300 mm)

\[ \sigma_v \] = effective vertical stress (MPa)

If CPT results are available, the settlement of a pile group bearing in sand can be estimated using:

\[ \rho' = \frac{qXI}{2q_c} \]  

(14.3.3.1-4)

where:

\[ q_c \] = average static cone resistance within a depth of \( X \) below the group (MPa)

Reference is made (SC10.7.2.4) to a procedure for estimating the lateral displacement of pile groups (e.g., Barker, et al, 1991). This method is based on various simplifying assumptions regarding uniformity of the soil profile and fixity of the pile in the cap. Therefore, the procedure can be used for preliminary evaluation and when the problem does not substantially violate the assumptions of the procedure. During the past decade, computer programs have become available for the lateral load-deflection
analysis of driven piles and drilled shafts based on the p-y curve method of analysis. For individual vertical piles, these programs include COM624 (Reese, 1984) for individual flexible vertical piles in level ground, LTBASE (Borden and Gabr, 1987) for individual flexible vertical piles in level and sloping ground, COM624P (Version 2) (Wang and Reese, 1993) for individual flexible and rigid vertical piles in level and sloping ground, and GROUP1 (Reese, et al, 1988) for a group of flexible vertical and battered piles in level ground can be used.

14.3.3.2 TOLERABLE MOVEMENT CRITERIA

The tolerable axial and lateral movement of driven piles should be established using the structural criteria described for shallow foundations. In some cases, the tolerable lateral movement is fixed at a small displacement (e.g., \#12 mm based on observed acceptable performance and engineering judgment) to ensure acceptable structure performance.

14.3.4 Resistance at the Strength Limit State

The resistance of driven pile foundations at the strength limit state should include consideration of:

• bearing resistance,
• uplift resistance,
• punching of a pile group through a strong layer into a weaker underlying layer,
• structural resistance of the piles under vertical and lateral loads, and
• anticipated damage from pile driving, if any.

Large displacements are typically required to cause passive failure of the soil in response to lateral loading of piles. Therefore, lateral resistance of the supporting soil need not be evaluated as long as tolerable movement criteria are met.

14.3.4.1 RESISTANCE FACTORS

Resistance factors for the geotechnical design of driven pile foundations are presented in Table 14.3.4.2-1. A majority of the resistance factors in the table are based on design procedures which are commonly used for allowable stress design (ASD) of axially-loaded piles (i.e., α, β and λ methods). Development of the resistance factors was based primarily on the use of reliability theory where a statistically sufficient source of information was
available. Where sufficient information was not available, selection of resistance factors was based on calibration with working stress designs and engineering judgment.

### 14.3.4.2 AXIAL LOADING

The factored bearing resistance of piles subjected to axial loading, \( \varphi_R \), can be determined as:

\[
Q_r = \varphi Q_n = \varphi_q Q_p + \varphi_s Q_s
\]

(14.3.4.2-1)

for which:

\[
Q_p = q_p A_p \quad \text{(14.3.4.2-2)}
\]

\[
Q_s = q_s A_s \quad \text{(14.3.4.2-3)}
\]

where:

\[\varphi_q, \varphi_s = \text{resistance factors for tip resistance and side resistance in Table 14.2.4.1-1 (dim)}\]

\[Q_p, Q_s = \text{pile tip and side resistance (N)}\]

\[q_p, q_s = \text{unit tip and side resistance (N)}\]

\[A_p, A_s = \text{area of the pile tip and side surface (mm}^2\text{)}\]

A further reduction in \( P_n \) for piles should be considered when pile driving difficulty is expected. In the past, a reduction multiplier factor of about 0.875 was considered when moderate driving difficulty was expected, and a factor of about 0.75 was considered when difficult driving was expected. For further details see Davisson (1983).
Using Equation 14.3.4.2-1, LRFD provides procedures for estimating the axial capacity of driven piles in soil using semi-empirical methods and in-situ testing, and for piles bearing on or in rock. When using Equation 14.3.4.2-1, it should be recognized that the pile capacity is comprised of components resulting from side resistance and tip resistance. Thus, for point-bearing piles, the

---

**Table 14.3.4.1-1 - Resistance Factors for Geotechnical Strength Limit State in Axially Loaded Piles**

(Tables S10.5.4-2 in LRFD)

<table>
<thead>
<tr>
<th>METHOD/SOIL/CONDITION</th>
<th>RESISTANCE FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ultimate Bearing Resistance of Single Piles</strong></td>
<td></td>
</tr>
<tr>
<td>Skin Friction: Clay</td>
<td></td>
</tr>
<tr>
<td>(\alpha)-method (Tomlinson, 1987)</td>
<td>0.70</td>
</tr>
<tr>
<td>(\beta)-method (Esrig &amp; Kirby, 1979)</td>
<td>0.50</td>
</tr>
<tr>
<td>(\lambda)-method (Vijayvergiya &amp; Focht, 1972)</td>
<td>0.50</td>
</tr>
<tr>
<td>End Bearing: Clay and Rock</td>
<td></td>
</tr>
<tr>
<td>Clay (Skempton, 1951)</td>
<td>0.70</td>
</tr>
<tr>
<td>Rock (Canadian Geotech. Society, 1985)</td>
<td>0.50</td>
</tr>
<tr>
<td>Skin Friction and End Bearing: Sand</td>
<td></td>
</tr>
<tr>
<td>SPT-method</td>
<td>0.45</td>
</tr>
<tr>
<td>CPT-method</td>
<td>0.55</td>
</tr>
<tr>
<td>Skin Friction and End Bearing: All Soils</td>
<td></td>
</tr>
<tr>
<td>Load Test</td>
<td>0.80</td>
</tr>
<tr>
<td>Pile Driving Analyzer</td>
<td>0.70</td>
</tr>
<tr>
<td><strong>Block Failure</strong></td>
<td></td>
</tr>
<tr>
<td>Clay</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Uplift Resistance of Single Piles</strong></td>
<td></td>
</tr>
<tr>
<td>(\alpha)-method</td>
<td>0.60</td>
</tr>
<tr>
<td>(\beta)-method</td>
<td>0.40</td>
</tr>
<tr>
<td>(\lambda)-method</td>
<td>0.45</td>
</tr>
<tr>
<td>SPT-method</td>
<td>0.35</td>
</tr>
<tr>
<td>CPT-method</td>
<td>0.45</td>
</tr>
<tr>
<td>Load Test</td>
<td>0.80</td>
</tr>
<tr>
<td><strong>Group Uplift Resistance</strong></td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td>0.55</td>
</tr>
<tr>
<td>Clay</td>
<td>0.55</td>
</tr>
</tbody>
</table>
tip resistance is the predominant source of pile capacity, and for friction piles, side resistance is the predominate source of pile capacity. For combination point-bearing and friction piles, pile capacity can be estimated by adding the relative contributions from tip and side resistance. It should be noted, however, that because the maximum values of side and tip resistance are mobilized at different displacements, care should be taken in using the numerical sum of the individual resistances for resistances less than \( Q_N \). Additionally, because the values of \( \phi_{qp} \) and \( \phi_{qs} \) depend on the method used to estimate the pile bearing resistance, the values of \( \phi_{qp} \) and \( \phi_{qs} \) will usually differ.

The semi-empirical methods presented in LRFD include the \( \alpha \)-method which is applicable for estimating \( q_s \) for displacement and non-displacement piles in cohesive soils, the \( \beta \)-method which is applicable for estimating \( q_s \) for displacement piles in normally to lightly overconsolidated cohesive soils, and the \( \lambda \)-method which is applicable for estimating \( q_s \) for displacement piles in cohesive soils. The in-situ test methods presented in LRFD include procedures for estimating \( q_s \) and \( q_p \) in cohesionless soils based on SPT and CPT test results, and \( q_p \) in rock based on information from coring to define the spacing and width of fractures and joints, and laboratory testing to determine the compressive strength of intact rock core.

As an example of procedures presented in LRFD to estimate the factored bearing resistance, \( Q_r \), of an axially-loaded driven pile foundation, the procedure presented for a pile in cohesive soil is presented herein. Using the \( \alpha \)-method, the nominal unit shaft resistance, \( q_{ns} \), can be determined as:

\[
q_s = \alpha S_u
\]  
(14.3.4.2-4)

where:

\[
\begin{align*}
S_u & = \text{mean undrained shear strength along the length of pile (MPa)} \\
\alpha & = \text{adhesion factor applied to } S_u \text{ (dim)}
\end{align*}
\]

The value of \( \alpha \) can be determined using Figure 14.3.4.2-1 which shows how \( \alpha \) varies, depending on the installation condition, pile length in cohesive soil and \( S_u \).
The nominal unit tip resistance, $q_p$, can be determined as:

$$q_p = 9S_u$$  \hspace{1cm} (14.3.4.2-5)

where:

$S_u$ = undrained shear strength of clay within 2 to 4 diameters of the pile tip (MPa)

The specification also provides guidance for determining the factored pile capacity based on the results of static pile load testing and dynamic monitoring of piles during pile driving operations.

The Specification also provides guidance for estimating pile capacity based on the results of static pile load testing and dynamic monitoring of piles during pile driving operations.
14.3.4.3 LATERAL LOADING

Most pile foundations supporting highway structures are subjected to lateral loading due to earth and water pressures, wind, earthquake and live loads. For bridges, the axial load is usually greater than the lateral load, while for structures such as noise walls and overhead sign structures, the lateral loads usually exceed the axial loads.

Pile foundations must be designed to resist lateral loads without structural failure of the pile (i.e., without reaching the strength limit state), and without deflecting excessively (i.e., without exceeding the tolerable lateral movement at the service limit state.) The analysis of a laterally-loaded pile or pile group is a complex soil-structure interaction problem. If the foundation materials are homogeneous (e.g., uniform cohesionless or cohesive soil deposit) and the pile head is fixed, simplified methods of analysis (e.g., Barker, et al, 1991) can be used. However, because the foundation loading conditions are usually more complex and the fixity of the pile is intermediate between the fixed- and free-head case, the analysis of laterally loaded piles and pile groups is usually accomplished using computer programs based on the p-y curve method of lateral load-deflection analysis. These programs include COM624 (Reese, 1984) for individual vertical piles in level ground, LTBASE (Borden and Gabr, 1987) for individual vertical piles in level and sloping ground, and GROUP1 (Reese, et al, 1988) for a group of vertical and battered piles in level ground can be used. Each of these methods of analysis incorporate p-y curves which model the nonlinear lateral load-deflection behavior for each soil type along the length of the pile. For pile groups, the effect of closely-spaced piles (i.e., where the center-to-center (CTC) spacing between piles is less than about 6 pile diameters) can be considered in the analyses by modifying the p-y relationships using efficiency relationships.

Design of laterally-loaded piles using the above methods can be performed using the following procedure:

1. For the pile section selected based on the service and strength limit state design for axial loading, determine the maximum lateral groundline deflection and the maximum factored moment for an individual pile at the strength limit state using an appropriate method of analysis and factored axial and lateral loads.

2. If the lateral groundline deflection exceeds a tolerable deformation or the maximum factored moment exceeds the factored moment capacity of the pile obtained from Sections 5, 6 or 8, select a new pile section and repeat Step 1.
3. If neither the lateral groundline deflection nor the factored moment criteria in Step 2 is exceeded, compute the maximum lateral groundline lateral deflection for the pile at the service limit state using an appropriate method of analysis and unfactored axial and lateral loads.

4. If the lateral groundline deflection exceeds the tolerable movement criteria, select a new trial section and repeat Step 1.

5. If the lateral groundline deflection does not exceed the tolerable movement criteria, the pile section is acceptable for the design loads.

For these analyses, lateral load-deflection relationships used to determine deflections should be not be factored, because these relationships are full-range and model non-linear behavior up to ultimate strength of the soil.

14.3.4.4 BATTER PILES

If the lateral loads acting on a foundation are large, batter piles are often used because they are more effective than vertical piles in transmitting lateral load into soil. Batter piles are more efficient in this regard because they transmit lateral loads predominately as an axial force into the soil, and because piles are substantially stiffer in axial loading than in lateral loading. In some instances, however, battered piles should not be used. These applications include:

- Installations where negative loading or downdrag forces can develop and cause bending, and
- applications in areas of high seismicity (i.e., Seismic Zones 3 and 4) where high lateral forces from earthquake loading can result in local overstressing of piles near the cap/batter pile contact, and punching of piles through the cap.

14.3.4.5 GROUP BEHAVIOR

The design of pile groups is similar to the design of an individual pile, such that the nominal resistance of the pile group, $Q_r$, is:

$$Q_r = \phi Q_n = \phi_g Q_g$$  \hspace{1cm} (14.3.4.5-1)

where:

$Q_g$ = nominal resistance of the group (N)
φₙ = group resistance factor from Table 14.2.4.1-1

In cohesive soils, the axial load resistance of a pile group may be less than the sum of the resistance of the individual piles due to overlapping of zones of shear deformation in the soil surrounding the piles. In stiff cohesive soils, there is no loss in resistance due to group effects. However, in soft cohesive soils where the pile cap is not in contact with the underlying soil, the resistance of a pile group will be less than the cumulative resistance of an equal number of individual piles if the CTC spacing between piles is less than six pile diameters. For closer pile spacings, the resistance of each pile is multiplied by an efficiency factor, η, such that:

\[ η = \begin{cases} 
0.65 & \text{for CTC spacing of 2.5 diameters} \\
1.0 & \text{for CTC spacing of 6.0 diameters.} 
\end{cases} \]

For intermediate spacings, the value of η can be determined by linear interpolation.

The value of \( Q_φ \) for a group of piles in cohesive soil should be determined as the lesser of:

- the sum of the modified individual resistances of each pile in the group, or
- the resistance of an equivalent pier consisting of the piles and the block of soil bounded within the piles.

For the equivalent pier method:

- the peak shear strength of the soil should be used to determine the side resistance,
- the total base area of the pier should be used to determine the end bearing resistance, and
- any resistance provided by the cap should be ignored.

The resistance factor for block failure should be used to determine the factored resistance of the pile group determined on the equivalent pier basis, regardless of whether the pile cap is or is not in contact with the ground. If the resistance of an equivalent number of individual piles is adjusted by η, the resistance factor for a single pile in Table S10.5.4-2 should be used to determine the factored resistance of the pile group. The commentary to S10.7.3.10.2 provides guidance regarding determination of \( Q_φ \) using the equivalent pier method.
For a pile group in cohesionless soil, the value of $\eta$ is 1.0, regardless of the spacing of piles or contact between the pile cap and the ground. No reduction in capacity is used for a pile group in sand due to the increase in soil density that usually occurs when piles are driven in sand. For this case, the resistance factor for axial capacity of a single pile in Table 10.5.4-2 should be used to determine the factored resistance of the pile group.

If a pile group is embedded in a stiff soil deposit overlying a weaker deposit, the potential exists for a punching failure of the pile group through the stiff soil into the soft soil. In addition, the potential for settlements in the soft layer must be evaluated. If the distance between the pile tips of the group and the top of the weaker layer is greater than 10 times the width of the group, $Q_g$ equals the value based on the soil properties into which the group is embedded. If the distance is less than ten times the width of the group, $Q_g$ can be determined as:

$$Q_g' = (Q_g)_0 \% \left[ \frac{(Q_g)_1 \& (Q_g)_0}{10B} \right] H < (Q_g)_1 \quad (14.3.4.5-2)$$

where:

- $(Q_g)_0$ = nominal group capacity for the weaker layer (N)
- $(Q_g)_1$ = nominal group capacity for the stronger layer (N)
- $B$ = pile group width (mm)
- $H$ = distance between pile tips and weaker layer (mm)

14.3.4.6 STRUCTURAL DESIGN

In addition to meeting the requirements for the structural design of piles in accordance with Sections 5, 6 and 8 of LRFD, the potential for buckling should be evaluated when piles extend through air or water before achieving lateral fixity at some depth into the ground. The provisions for stability of compression members in Sections 5, 6 and 8 of LRFD require the determination of the depth to fixity to estimate the equivalent unsupported length of the pile. For these cases, the depth to fixity, $D_f$ in mm, can be determined as:

- for piles in cohesive soil:

$$D_f' = 1.4 \left[ \frac{E_p T_p}{E_b} \right]^{0.25} \quad (14.3.4.6-1)$$
• for piles in cohesionless soil:

\[ D_f' = 1.8 \left[ \frac{E_p I_p}{n_h} \right]^{0.20} \]  

(14.3.4.6-2)

where:

\[ E_p \] = modulus of elasticity of the pile section (MPa)

\[ I_p \] = moment of inertia of the pile section (mm$^4$)

\[ E_s \] = soil modulus for cohesive soils = 67$S_u$ (MPa)

\[ S_u \] = undrained shear strength of cohesive soil (MPa)

\[ n_h \] = rate of increase of soil modulus with depth for cohesionless soil (MPa/mm)  

(Table S10.7.4.2-1)

If the spacing between piles is less than eight pile diameters, the modulus of cohesive soil should be reduced in the previous relationship. If the spacing between piles is three pile diameters, the soil modulus should be reduced by 25%, and the reduction for spacings between eight and three piles diameters can be determined by interpolation.

In Equations 1 and 2, the loading condition has been assumed to be axial load only and the piles are assumed to be fixed at their ends. Since the equations give depth to fixity from the ground line, the Engineer must determine the boundary conditions at the top of the pile to determine the total unbraced length of the pile. If other loading or pile tip conditions exist, see Davisson and Robinson (1965).
REFERENCES


Davisson, M. T., and Robinson, K. E., "Bending and Buckling of Partially Embedded Piles", Proc. Sixth Int. Conf. S. M. and F. E., Montreal, Canada, University of Toronto Press, pp. 243-246, 1965


LECTURE 15 - INTRODUCTION TO WALLS

15.1 OBJECTIVE OF LESSON

The purpose of this lesson is to introduce the design of conventional walls and abutments, anchored walls and mechanically-stabilized earth walls required by the LRFD Specification. The lesson identifies general considerations which should be part of wall design, outlines the design procedure, and presents requirements for the geotechnical and structural design of walls at the service and strength limit states.

15.2 CONVENTIONAL RETAINING WALL AND ABUTMENT DESIGN

15.2.1 General Design Considerations

In addition to determining the design of the wall section to meet the service and strength limit states criteria for applied loads, the design of conventional walls and abutments requires consideration of factors which can affect wall and abutment performance, including:

• quality and drainage of backfill materials (S11.6.1.1)

• integral abutments and the effects of constraining wall movements (S11.6.1.2)

• load effects in abutments due to the weight of earth overlying the rear face or base of the abutment (S11.6.1.3)

• requirements for placement of construction and expansion joints (S11.6.1.5)

• structural and geometric design of wingwalls and cantilevered walls (S11.6.1.4)

• scour of foundations subjected to flowing water and subsurface erosion or piping due to elevated water levels in poorly-drained backfill materials (S11.6.3.5)

• seismic loading (S11.6.5 and Appendix S11.11)

15.2.2 Design Procedure

The design procedure for conventional walls and abutments involves the following steps:
1. Select wall type and preliminary wall proportions considering grade separation requirements, site constraints and foundation conditions.

2. Develop design foundation profile including soil and rock strata layering, engineering properties of foundation strata, groundwater level and problem conditions.

3. Estimate applied loads and earth pressure for strength and service limit states analyses and select resistance factors for conditions analyzed.

4. Check stability criteria along the base of the wall at the strength limit state for: (a) location of factored resultant force; (b) factored bearing resistance, and (c) factored sliding resistance; and revise wall proportions as necessary to meet stability criteria.

5. Design deep foundation for wall if acceptable factored bearing and sliding resistance cannot be developed using a wall base of reasonable width.

6. Check overall stability against deep-seated foundation failure.

7. Check vertical and horizontal wall displacements with tolerable movement criteria at service limit strength.

8. Design structural elements according to the provisions of Sections 5, 6 and/or 8 of LRFD.

9. Compare cost of wall design with other wall systems.

   Table 15.2.2-1 summarizes the strength and service limits state that must be evaluated for the design of conventional retaining walls and abutments.

   Walls should also be investigated for the extreme event limit state using a resistance factor of 1.0 unless specified otherwise.
Table 15.2.2-1 - Strength and Service Limit States for Design of Conventional Retaining Walls and Abutments

<table>
<thead>
<tr>
<th>Design Consideration</th>
<th>Strength Limit State</th>
<th>Service Limit State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location of Base Resultant Force</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Bearing Resistance</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Sliding Resistance</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Overall Stability</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Settlement and Horizontal Movement</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Structural Capacity of Wall Elements</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

15.2.3 Movement at the Service Limit State

15.2.3.1 ANALYSIS OF WALL DISPLACEMENTS

The Specification requires that the displacement of conventional walls and abutments be evaluated at the service limit state for all applicable load combinations. Usually this will be the Service I Load Combination from Table S3.4.1-1 in LRFD. Vertical and lateral wall movements can be estimated using the procedures described in Section 10 of LRFD for foundations and considering, as applicable, differential movements along the base of walls supported on footings due to variations in contact pressure. Lateral movements of walls on shallow foundations can be estimated assuming the wall displaces as a rigid body due to the effects of earth loads and differential settlements along the base of the wall.

15.2.3.2 TOLERABLE MOVEMENT CRITERIA

The tolerable movement criteria for abutments should follow the guidance provided in Section 14.2.3.2 for spread footing foundations. For other retaining walls, tolerable movement criteria should be developed with consideration of the function and type of wall, anticipated service life and consequences of unacceptable movements (e.g., affect of wall movements on adjacent facilities).

15.2.4 Resistance at the Strength Limit State

Abutments and conventional retaining walls must be proportioned to ensure adequate resistance at the strength limit state based on consideration of:
• eccentricity of the base resultant force due to lateral and inclined loading
• bearing resistance
• sliding resistance to lateral and inclined loading
• overall stability against deep-seated failure of foundation materials below the structure
• structural resistance of wall components

These failure modes are illustrated in Figure 15.2.4-1.

Figure 15.2.4-1 - Failure Modes for Retaining Walls

15.2.4.1 RESISTANCE FACTORS

Resistance factors for the design of conventional walls and abutments supported on shallow foundations are the same as those used for the design of spread footings presented in Table 14.2.4.1-1. If the stability criteria for wall design cannot be achieved using a spread footing for foundation support of the wall, deep foundation
design should follow the provisions of S10.7 and S10.8 for driven pile and drilled shaft foundations, respectively. The location of the resultant force along the wall base was developed by comparison with current allowable stress design procedures for walls supported on soil and rock. The provisions of Sections 5, 6 and/or 8 of LRFD should be followed in evaluating the structural resistance of wall components.

15.2.4.2 LOAD FACTORS

Article S3.4.1 of LRFD specifies that certain permanent loads, including earth loads, be factored using maximum and minimum load factors $\gamma_p$ as required in Table 15.2.4.2-1.

Table 15.2.4.2-1 - Load Factors for Permanent Structure and Earth Loads for Walls

<table>
<thead>
<tr>
<th>Type of Load</th>
<th>Load Factor Maximum</th>
<th>Load Factor Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC: Components and Attachments</td>
<td>1.25</td>
<td>0.90</td>
</tr>
<tr>
<td>EH: Horizontal Earth Pressure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Active Pressure</td>
<td>1.50</td>
<td>0.90</td>
</tr>
<tr>
<td>• At-Rest Pressure</td>
<td>1.35</td>
<td>0.90</td>
</tr>
<tr>
<td>EV: Vertical Earth Pressure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Retaining Structure</td>
<td>1.35</td>
<td>1.00</td>
</tr>
<tr>
<td>ES: Earth Surcharge</td>
<td>1.50</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Criteria in S3.4.1 require the following:

- Load factors shall be selected to produce the total extreme factored force effect, and for each combination, both positive and negative extremes shall be investigated.

- For permanent force effects, the load factor which produces the more critical combination shall be selected from Table 15.2.4.2-1.

- For load combinations where one force effect decreases the effect of another force (e.g., top/sidewall design of box culvert), the minimum value shall be applied to the load that reduces the force effect.

- If a permanent load increases the stability or load carrying capacity of a structure component (e.g., load from soil backfill on heel of wall), the minimum value for that permanent load shall also be investigated.
Applying these criteria for the evaluation of the sliding resistance of walls:

- The vertical earth load on the rear of a cantilevered retaining wall would be multiplied by $\gamma_{p\text{min}} (1.00)$ and the weight of the structure would be multiplied by $\gamma_{p\text{min}} (0.90)$ because these forces result in an increase in the contact stress (and shear strength) at the base of the wall and foundation.

- The horizontal earth load on a cantilevered retaining wall would be multiplied by $\gamma_{p\text{max}} (1.50)$ for an active earth pressure distribution because the force results in a more critical sliding force at the base of the wall.

Similarly, the values of $\gamma_{p\text{max}}$ for structure weight (1.25), vertical earth load (1.35) and horizontal active earth pressure (1.50) would represent the critical load combination for an evaluation of foundation bearing resistance.

15.2.4.3 OVERALL STABILITY

The overall stability of an abutment or retaining wall, retained slope and foundation materials should be evaluated for all walls using limit equilibrium methods of slope stability analysis. This analysis applies to deep failure of the soil mass, e.g., slope stability, not the stability of the wall itself which is covered in Article 15.2.4.4. These analyses should be conducted using a load factor of 1.35 for all earth, water and externally-applied loads used in the analysis, and a resistance factor of 0.85 (see Table S10.5.4-1 in LRFD) should be applied to the shear strength of soil and rock materials. Because resistance to slope instability can be affected by geologic features (e.g., bedding planes and fracturing), the anisotropic strength behavior of soils and layered rock and rate of loading effects, it is important that the strength properties and method of analysis correctly represent these conditions in the analysis. Although the potential for overall slope failure should be considered for all walls and abutments, slope stability is of particular concern for walls which are supported on weak foundation materials and/or are backfilled with poor draining soils that result in an elevated water level behind the wall.

15.2.4.4 LOCATION OF RESULTANT FORCE

Allowable stress methods for the design of conventional walls and abutments require evaluation of overturning as a potential failure mode. For this condition, walls on soil are designed for a minimum factor of safety (FS) of 2.00, and walls on rock are designed for a minimum FS of 1.75. At the strength limit state, LRFD evaluates overturning based on the location of the bearing pressure resultant as follows:
• For foundations on soil, the resultant of the factored reaction force shall be within the middle one-half of the base of the wall footing.

• For foundations on rock, the resultant of the factored reaction force shall be within the middle three-fourths of the base of the wall footing.

The location of the resultant force within the specified locations considers the use of a plastic bearing pressure distribution at the strength limit state.

The effect of earthquake is investigated using the extreme event limit state of Table S3.4.1-1 with resistance factors $\varphi = 1.0$ and an accepted methodology.

For foundations on soil, the location of the resultant of the reaction forces must be within the middle one-third of the base for $\gamma_{EQ} = 0.0$ and within the middle one-half of the base for $\gamma_{EQ} = 1.0$.

For foundations on rock, the location of the resultant of the reaction forces must be within the middle one-half of the base for $\gamma_{EQ} = 0.0$ and within the middle two-thirds of the base for $\gamma_{EQ} = 1.0$.

For values of $\gamma_{EQ}$ between 0.0 and 1.0, the restrictions of the location of the resultant are obtained by linear interpolation of the values given in this Article.

15.2.4.5 BEARING RESISTANCE

The bearing resistance of foundations supporting walls and abutments is determined following the provisions of S10 for the design of foundations. Care must be taken to perform these analyses for all appropriate load combinations.

15.2.4.6 SLIDING RESISTANCE

The sliding resistance of foundations supporting walls and abutments is determined following the provisions of S10 for the design of foundations. Care must be taken to perform these analyses for all appropriate load combinations.

15.2.4.7 CONTINUATION OF RETAINING WALL DESIGN EXAMPLE

The loads and load combinations required for each limit state to be investigated were calculated in Section 4.3.9. This section continues with foundation design aspects.
Step 4: Calculate the Settlement of the Retaining Wall on its Cohesive Foundation

Assume embankment construction has been performed earlier and that consolidation settlement from the embankment loading beneath the wall has already occurred.

Divide the 6 m of cohesive soil below the wall foundation into four layers with the following thicknesses:

\[ H_1 = 1 \text{ m} \]
\[ H_2 = 1 \text{ m} \]
\[ H_3 = 2 \text{ m} \]
\[ H_4 = 2 \text{ m} \]

Depth of footing below the existing ground surface is 1 m.

The depth to the center of each layer from the existing ground surface is

\[ d_1 = 1.5 \text{ m} \]
\[ d_2 = 2.5 \text{ m} \]
\[ d_3 = 4 \text{ m} \]
\[ d_4 = 6 \text{ m} \]

(A) Calculate the existing overburden pressure at the center of each layer - Using effective unit density:

\[ P_{O1} = (d_1)(760)g/(10^6) = 0.0112 \text{ MPa} \]
\[ P_{O2} = (d_2)(760)g/(10^6) = 0.0186 \text{ MPa} \]
\[ P_{O3} = (d_3)(760)g/(10^6) = 0.0298 \text{ MPa} \]
\[ P_{O4} = (d_4)(760)g/(10^6) = 0.0447 \text{ MPa} \]

(B) Calculate Increase in vertical pressure resulting from loading of the wall

Investigate vertical movement at the service limit state SA10.6.2.1.

Because the underlying material is cohesive, the determination of settlement is based on the permanent loads. Using Service I loading from Table 4.3.9-5, the total vertical load for calculation of settlement is 281 379 -34 167 = 247 212 N/m.

Considering an evenly distributed loading over the width of the wall foundation (3 m), the increase in pressure at the base of the footing is \[ P_v = (247 212)/(3x10^6) = 0.0824 \text{ MPa} \].
Table 15.2.4.7-1 - Calculation of Overburden Pressures

<table>
<thead>
<tr>
<th>Layer i</th>
<th>((d_{i-1})/B)</th>
<th>(P_v/P_v)</th>
<th>(P_{vi}) MPa</th>
<th>(P_{Oi}) MPa</th>
<th>(H) m</th>
<th>(H_i \log A = 1 + P_{vi}/P_{Oi})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.17</td>
<td>0.95</td>
<td>0.0783</td>
<td>0.0112</td>
<td>1</td>
<td>0.9025</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>0.80</td>
<td>0.0659</td>
<td>0.0186</td>
<td>1</td>
<td>0.6574</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.55</td>
<td>0.0453</td>
<td>0.0298</td>
<td>2</td>
<td>0.7026</td>
</tr>
<tr>
<td>4</td>
<td>1.67</td>
<td>0.34</td>
<td>0.0280</td>
<td>0.0447</td>
<td>2</td>
<td>0.5124</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.7749</td>
</tr>
</tbody>
</table>

where:

\[ B = 3 \text{ m} \]

\(d_{i-1}\) is the depth to the center of layer \(i\) below the footing

\(P_v/P_v\) is obtained using the Boussinesq stress contours at the center of a continuous foundation.

(C) Estimate the settlement using equations in Das, 1984

\[ S = 3 \left[ H_i \left( \log(P_{Oi} + P_{vi}) / P_{Oi} \right) \right] C_r / (1 + e_o) \]

with \(C_r = 0.012\) and \(e_o = 0.63\)

\[ S = 20 \text{ mm} \]

**Step 5: Check Eccentricity**

Table 15.2.4.7-2 - Summary for Eccentricity Check

<table>
<thead>
<tr>
<th>Group/Item Units</th>
<th>(V_{TOT}) N/m</th>
<th>(H_{TOT}) N/m</th>
<th>(M_v) N-m/m</th>
<th>(M_h) N-m/m</th>
<th>(X_o) m</th>
<th>(e_B) m</th>
<th>(e_{max}) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength I-a</td>
<td>299,235</td>
<td>145,766</td>
<td>536,649</td>
<td>313,206</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>385,759</td>
<td>145,766</td>
<td>685,656</td>
<td>313,206</td>
<td>0.97</td>
<td>0.53</td>
<td>0.75</td>
</tr>
<tr>
<td>Strength IV</td>
<td>345,390</td>
<td>102,416</td>
<td>587,747</td>
<td>204,833</td>
<td>1.11</td>
<td>0.39</td>
<td>0.75</td>
</tr>
<tr>
<td>Service I</td>
<td>281,379</td>
<td>93,049</td>
<td>494,068</td>
<td>198,483</td>
<td>1.05</td>
<td>0.45</td>
<td>0.75</td>
</tr>
</tbody>
</table>

where:

\[ X_o = \text{location of the resultant} = (M_v - M_h) / V_{TOT} \]
\( e_B = \text{eccentricity} = 0.5B - X_o \)

Using the provisions of either Article S10.6.3.1.5 or S11.6.3.3, \( e_{\text{max}} = B/4 \)

For all cases, \( e_B \) is less than or equal to \( e_{\text{max}} \), therefore, the design is adequate with regards to eccentricity.

The adequacy for bearing capacity is developed based on a rectangular distribution of soil pressure as indicated in S10.6.3.1.5, a figure which is repeated below.

![Figure 15.2.4.7-1 - Reduced Footing Dimensions](image)

**Step 6: Check Bearing Capacity**

(A) Estimate the Bearing Pressures

<table>
<thead>
<tr>
<th>Group/Item Units</th>
<th>( V_{\text{TOT}} ) N/m</th>
<th>( X_o ) m</th>
<th>( e_B ) m</th>
<th>( q_{\text{max}} ) MPa</th>
<th>( q_{\text{min}} ) MPa</th>
<th>( q_{\text{uniform}} ) MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength I-a</td>
<td>299,235</td>
<td>0.75</td>
<td>0.75</td>
<td>0.2672</td>
<td>0</td>
<td>0.2004</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>385,759</td>
<td>0.97</td>
<td>0.53</td>
<td>0.2664</td>
<td>0</td>
<td>0.1998</td>
</tr>
<tr>
<td>Strength IV</td>
<td>345,390</td>
<td>1.11</td>
<td>0.39</td>
<td>0.2052</td>
<td>0.0250</td>
<td>0.1556</td>
</tr>
<tr>
<td>Service I</td>
<td>281,379</td>
<td>1.05</td>
<td>0.45</td>
<td>0.1781</td>
<td>0.0095</td>
<td>0.1339</td>
</tr>
</tbody>
</table>

For a rectangular distribution use \( B = B' \) and \( L = L' \):
L' = L
q = \( \frac{V_{TOT}}{(L' B')} \)
L = wall length
B' = B - 2 \( e_B \)

For a triangular base pressure distribution:

\[ q_{\text{max}} = \text{pressure at toe} = 2 \frac{V_{TOT}}{(3X_o)} \]

\[ q_{\text{uniform}} = \text{average pressure for an assumed rectangular distribution} = \frac{V_{TOT}}{(2X_o)} \]

For a trapezoidal base pressure distribution using full-width, B = 3 m:

\[ q_{\text{max}} = \text{pressure at toe} = \left( \frac{V_{TOT}}{B} \right) + \left[ \frac{(6)(V_{TOT})(e_B)}{B^2} \right] \]

\[ q_{\text{min}} = \text{pressure at heel} = \left( \frac{V_{TOT}}{B} \right) - \left[ \frac{(6)(V_{TOT})(e_B)}{B^2} \right] \]

\[ q_{\text{uniform}} = \text{average pressure for an assumed rectangular distribution} = \frac{0.5 V_{TOT}}{X_o} \]

(B) Estimate the Bearing Capacity

The factored resistance at the strength limit state is defined by the following:

\[ q_r = \varphi \cdot q_{\text{ult}} \]

where:

\[ \varphi = \text{resistance factor} = 0.60 \]

From Table S10.5.4-1 for clay using the rational method:

\[ q_{\text{ult}} = \text{nominal bearing resistance} \]
From S10.6.3.1.2b for the case with saturated clays

\[ q_{ult} = c N_{cm} + g Y_2 D_f N_{qm} (10^{-9}) \]

where:

\[ c = 0.15 \text{ MPa} \]

\( N_{cm} \) and \( N_{qm} \) are modified bearing capacity factor which are functions of shape, embedment and load inclination and are defined as follows:

From Equation 14.2.4.2-2 using \( B = B' \) and Strength 1-A

\[ N_{cm} = 5(1+0.2D_f/B)(1+0.2B/L)(1-1.3H/V) \]

\[ N_{cm} = 3.324 \]

where \( H \) and \( V \) are the unfactored horizontal and vertical loads, respectively, and \( L = 50 \text{ m} \) (assumed)

From Equation 14.2.4.2-7

\[ N_{qm} = 1.0 \text{ for cohesive soil} \]

The shape factor \( (s_q) \), the soil compressibility factor \( (c_q) \) and the embedment depth factor \( (d_q) \) have all been assumed to be 1.0.

\[ q_{ult} = 0.15(3.324) + 9.81(1760)(1000)(1.0)(10^{-9}) \]

\[ q_{ult} = 0.515 \text{ MPa} \]

\[ q_{fr} = 0.309 \text{ MPa} \]

The estimated bearing capacity of the soil exceeds the maximum and uniform bearing pressures of the wall foundation for the strength and service load groups analyzed.
Step 7: Check Sliding

Sliding of walls on clay is checked using Figure S10.5.4-1 which is repeated below as Figure 15.2.4.7-2.

\[ q_s = \text{unit shear resistance, equal to } S_u \text{ or } 0.5\sigma'_v, \text{ whichever is less} \]

\[ Q_r = \text{area under } q_s \text{ diagram (shaded area)} \]

\[ S_u = \text{undrained shear strength (MPa)} \]

\[ \sigma'_v = \text{vertical effective stress (MPa)} \]

Figure 15.2.4.7-2 - Procedure for Estimating Sliding Resistance for Walls on Clay

From Equation S10.6.3.3-1, the factored resistance against failure by sliding is taken as:

\[ Q_R (N) = \phi_r Q_r + \phi_{ep} Q_{ep} \]

with \( Q_r \) and \( Q_{ep} \) in N

Using S10.6.3.3:

\[ Q_r = \text{the lesser of the following} \]

\[ \cdot \text{cohesion of the clay} \]
one-half the normal stress on the interface between the footing and the soil where the cohesion $q_s = 0.1500$ MPa

Table 15.2.4.7-4 - Summary of Sliding Resistance

<table>
<thead>
<tr>
<th>Group/Item</th>
<th>$q_{\text{max}}/2$ MPa</th>
<th>$3(X_o)$ m</th>
<th>$Q_t$ N/m</th>
<th>$\varphi_t Q_t$ N/m</th>
<th>$H_{\text{TOT}}$ N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength I-a</td>
<td>0.1336</td>
<td>2.24</td>
<td>149,617</td>
<td>127,175</td>
<td>145,766</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>0.1332</td>
<td>2.90</td>
<td>192,880</td>
<td>163,948</td>
<td>145,766</td>
</tr>
<tr>
<td>Strength IV</td>
<td>0.1026</td>
<td>3.33</td>
<td>170,657</td>
<td>145,058</td>
<td>102,416</td>
</tr>
<tr>
<td>Service I</td>
<td>0.0891</td>
<td>3.15</td>
<td>140,331</td>
<td>119,281</td>
<td>93,049</td>
</tr>
</tbody>
</table>

Since $q_{\text{max}}/2$ is less than $q_s$ in all cases, $q_{\text{max}}/2$ is used in the calculation of $Q_t$:

$$Q_t = (0.5)(q_{\text{max}}/2)(3X_o)$$

$$\varphi_t = 0.85$$

Since the sliding resistance ($\varphi, Q_t$), calculated above, is less than the horizontal loading for group Strength I-a, the effect of the passive resistance must be considered to provide adequate resistance against sliding, S11.6.3.6.

Assuming that the 0.5 m of soil above the top of the footing may some day be removed, a depth of 0.5 m (thickness of the wall footing) will be used for calculation of the passive resistance.

Using Figure 5.3.9-3, presented in Step 1, since $\varphi = 0$, $K_p=1$

By Equation 5.22 - Das (1984), the total passive pressure per meter of wall length is

$$Q_{ep} = P_p = 0.5 \gamma_s g H^2 K_p + 2 cH/K_p \times 10^6$$

which reduces to

$$Q_{ep} = P_p = (0.5)(1760 g)(0.5^2) + (2)(0.15)(0.5) \times 10^6$$

$Q_{ep} = P_p = 152,158$ N/m

Table S10.5.4-1 gives $\varphi_{ep} = 0.50$ for passive earth pressure component of sliding resistance

Lecture - 15-14
$\varphi_{ep} Q_{ep} = 76,079 \text{ N/m}$

$Q_R = 127,175 + 76,079 = 203,254 \text{ N/m}$

From Table 15.2.4.7-4, the worst case for sliding is for Loading Group Strength I-a. Since $Q_R$ is greater than $H_{TOT} = 145,766$, adequate protection against sliding is provided.

15.2.4.8 STRUCTURAL DESIGN

The design of conventional walls and abutments must meet the requirements of Section S5 for the design of concrete structures. Barker, et al, (1991) provides the following guidance with regard the structural design of wall components:

- **Wall Stem**
  - The wall stem is designed as a cantilever supported above the base.
  - The wall stem of a counterfort (or buttress) can be designed as a horizontal fixed or continuous beam supported by counterforts or buttresses. The wall stem of these structures can also be designed as a plate supported in three sides by the base slab and counterforts (or buttresses), and free at the top.

- **Base Slab**
  - The base slab of a cantilevered wall is designed as a cantilever fixed at the wall.
  - The base slab of a counterfort or buttress wall is designed as a fixed or continuous beam spanning between the counterforts or buttresses.
  - The maximum bending moment for base slab design is taken at the face of the wall stem for the toe and the back of the wall stem for the heel.
  - The maximum design shear forces are taken at a distance equal to effective depth of the slab from the face of the wall stem for the toe section and at the back of the wall stem for the heel section.
• Counterforts and Buttresses

- Counterforts are designed as T-beams, whereas buttresses are designed as rectangular beams. In counterfort and buttress walls, tension reinforcement should be provided as should a combination of horizontal and vertical bars or stirrups at the junction of the counterfort or buttress with the wall stem.

15.3 ANCHORED RETAINING WALL DESIGN

15.3.1 General Design Considerations

As shown in Figure 15.3.1-1, anchored walls are comprised of anchors constructed using prestressing strand or steel bars, discrete vertical wall elements (i.e., usually rolled steel section) spaced approximately 2 to 3 m along the length of the wall and facing (i.e., timber, precast concrete lagging or shotcrete).

![Figure 15.3.1-1 - Anchored Wall Nomenclature](image)

Anchors are installed into drilled holes and extend through the wall face to a bearing plate supported on the vertical wall element or a wale between adjacent vertical wall elements. Because most anchored walls are constructed from the top down, wall construction proceeds by:

- Installing discrete vertical wall elements by driving or concreting into a predrilled hole
- Excavating to a level of between 2 to 4 m to install the upper level of anchors, and installing facing support, as required

Lecture - 15-16
• Drilling the anchor hole at an angle of about 10 to 30 degrees to the horizontal, and installing and grouting the anchor in the bond length

• Stressing and testing each anchor, locking off the anchor at the design load, and grouting the unbonded length if the anchor has grease and sheathing corrosion protection

• Repeating the excavation, anchor hole drilling, installation, grouting and testing, and facing installation for each anchor level to the base of the wall

• Constructing a wall facing in front of the wall for permanent construction.

The geometry of wall construction, shown in Figure 15.3.1-1, is typical for most anchored walls. The length of anchor is designed to provide an adequate bond length to resist loads on the wall and to extend a reasonable distance beyond the critical failure surface. Although the inclination of anchors is usually limited to maximum of about 30° to the horizontal to minimize axial loads on the vertical wall elements, steeper inclinations can be used to avoid interference with buried structures or to reach more suitable materials which provide a higher bond capacity. The vertical spacing of anchors can be optimized to provide a balanced design between the number of anchors and bending of the vertical wall element.

In addition to determining the design wall section to meet the service and strength limit states criteria for applied loads, the design of anchored walls requires consideration of factors which can affect wall and abutment performance, including:

• seismic loading (S11.8.6 and Appendix S11.1);

• corrosion protection (S11.8.7);

• anchor stressing and testing during construction (S11.8.8.1)

• drainage control to mitigate build up of water pressure behind the wall (S11.8.9)

15.3.2 Design Procedure

The design procedure for anchored walls involves the following steps:

1. Identify site constraints (e.g., inadequate right-of-way for anchor installation) that could affect anchor wall construction.
2. Develop design profile including: soil and rock type and layering, engineering properties of retained and foundation strata, groundwater level, and problem conditions (e.g., corrosion).

3. Estimate applied loads and earth pressure for strength and service limit states analyses and select resistance factors for conditions analyzed.

4. Determine number and location of anchors to resist pullout from factored earth and applied loads.

5. Select spacing of vertical wall elements and design elements to resist factored earth and applied loads between elements and the vertical component of the factored anchor loads according to the provisions of Sections 5 or 6 of LRFD, depending on the type of wall element.

6. Evaluate the resistance of the embedment zone against end bearing and passive earth failure.

7. Select the type and design the facing between vertical wall elements to resist the factored loads on the facing considering the effects of soil arching between vertical wall elements according to the provisions of Sections 5 or 8 of LRFD, depending on the type of facing selected.

8. Check vertical and horizontal wall displacements with tolerable movement criteria at service limit strength.

9. Check overall stability against deep-seated foundation failure.

10. Determine whether special construction control is required, such as preconstruction anchor stressing and testing or wall performance monitoring during construction.

11. Compare cost of wall design with other wall systems.

Table 15.3.2-1 summarizes the strength and service limits states that must be evaluated for the design of anchored walls.

Walls should also be investigated for the extreme event limit state using a resistance factor of 1.0 unless specified otherwise.
Table 15.3.2-1 - Strength and Service Limit States for Design of Anchored Walls

<table>
<thead>
<tr>
<th>Design Consideration</th>
<th>Strength Limit State</th>
<th>Service Limit State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anchor Pullout</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Structural Resistance of Anchor</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Structural Resistance of Vertical Wall Element</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Passive and Bearing Resistance of Embedment</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Structural Resistance of Facing Elements</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Overall Stability</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Settlement and Horizontal Movement</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

15.3.3 Movement at the Service Limit State

15.3.3.1 ANALYSIS OF WALL DISPLACEMENTS

The displacement of anchored walls shall be evaluated at the service limit state for all applicable load combinations using the Service I Load Combination from Table S3.4.1-1 in LRFD. Because the vertical and lateral displacement of anchored walls is a complex soil-structure interaction problem, deformation analyses can be performed using modified forms of beam on elastic foundation theory or finite element analyses. For many projects, however, these types of analyses are usually not warranted unless deformation-sensitive structures are in close proximity to the wall. As an alternative, for walls built using well proven anchored wall construction technology, Figure 15.3.3.1-1 can be used as a guide to estimate settlement of the ground surface behind anchored walls. Although not shown in the figure, the maximum lateral movement for walls supporting sand and stiff to hard clay soils usually does not exceed about 0.3% of the depth of excavation. It should be noted that the relationships in the figure do not include the effects of settlements caused by other construction activities (e.g., dewatering, ground heave at the base of the wall, or poor construction quality during anchor, or vertical wall element or facing installation).
CURVE I = sand
CURVE II = stiff to very hard clay
CURVE III = soft to medium clay - with high resistance to base heave
CURVE IV = soft to medium clay - with marginal resistance to base heave

Figure 15.3.3.1-1 - Settlement Profiles Behind Anchored Walls (Modified after Clough and O'Rourke 1990)

15.3.3.2 TOLERABLE MOVEMENT CRITERIA

The tolerable movement of the wall should be developed based on consideration of the effects of possible wall movements on other structures and facilities near the wall.

15.3.4 Resistance at the Strength Limit State

Anchored walls must be proportioned to ensure adequate resistance at the strength limit state based on consideration of:

- Pullout and structural failure of anchor due to imposed loads
- Passive and bearing resistance of soil or rock into which vertical wall element is embedded
- Structural capacity of vertical wall elements and facing
- Overall stability against deep-seated failure of foundation materials below the structure
15.3.4.1 RESISTANCE FACTORS

Resistance factors for the design of anchored walls are presented in Table 15.3.4.1-1. The resistance factors were developed based on engineering judgment and calibration with current allowable stress design procedures for anchored walls. The provisions of Sections 5, 6 and/or 8 of LRFD should be followed in evaluating the structural resistance of wall components.

15.3.4.2 ANCHOR PULLOUT

Prestressed anchors used for anchored wall construction must be designed to resist pullout of the bonded length of anchor in soil or rock. The minimum bonded length, L, is determined as:

\[
L = \frac{Q_u}{\varphi Q_a}
\]

(15.3.4.2-1)

where:

- \(L\) = bond length (mm)
- \(Q_u\) = factored anchor load from contributory area (N) based on a soil pressure diagram consistent with S3.11
- \(Q_a\) = unit resistance between grout and soil or rock in anchor bond zone (N/mm)
- \(\varphi\) = resistance factor from Table S11.5.6-1

Tables 15.3.4.2-1 and 15.3.4.2-2 provide conservative values of \(Q_a\) for anchors in soil and rock, respectively. These values are intended for preliminary design or evaluation of the feasibility of straight shaft anchors installed in small diameter holes which are grouted using low pressures. Pressure-grouted anchors usually achieve higher capacities. Because anchor capacity in soil and rock can be strongly influenced by the method of anchor hole advancement, hole diameter, anchor type, type of grout and grouting pressure, selection of the anchor type and determination of \(Q_a\) for final design should be made by the specialty geotechnical contractor selected for wall construction.

Several methods are available for determining the factored anchor load, \(Q_u\). In the proportional method, the top anchor row is assumed to support the tributary area of pressure between the top of the wall and the mid-point between the upper two anchor levels; and the bottom anchor row is assumed to support the pressure between the base of the wall and the mid-point between the two lowest anchor rows. Optionally, the embedded portion of the vertical wall element can be assumed to
support the pressure between the base of the exposed wall and the mid-point between the base and the lowest anchor row. For either option and intermediate anchor rows are assumed to support the pressures from the midway points between vertically adjacent anchor rows.

Table 15.3.4.1-1 - Resistance Factors for Anchored Walls

<table>
<thead>
<tr>
<th>WALL-TYPE AND CONDITION</th>
<th>RESISTANCE FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing resistance of vertical elements</td>
<td>S10.5 applies</td>
</tr>
<tr>
<td>Overturning</td>
<td>Passive resistance of vertical elements</td>
</tr>
<tr>
<td></td>
<td>• in soil</td>
</tr>
<tr>
<td></td>
<td>• in rock</td>
</tr>
<tr>
<td>Anchor pullout resistance</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Sand</td>
</tr>
<tr>
<td></td>
<td>correlation with SPT resistance-corrected for overburden pressure</td>
</tr>
<tr>
<td></td>
<td>pullout load tests</td>
</tr>
<tr>
<td></td>
<td>• Clay</td>
</tr>
<tr>
<td></td>
<td>correlation with unconfined compressive strength</td>
</tr>
<tr>
<td></td>
<td>using shear strength from lab tests</td>
</tr>
<tr>
<td></td>
<td>using shear strength from field tests</td>
</tr>
<tr>
<td></td>
<td>pullout load tests</td>
</tr>
<tr>
<td></td>
<td>• Rock</td>
</tr>
<tr>
<td></td>
<td>correlation with rock-type only</td>
</tr>
<tr>
<td></td>
<td>using minimum shear resistance measured in lab tests - soft rock only</td>
</tr>
<tr>
<td></td>
<td>laboratory rock-grout bond tests</td>
</tr>
<tr>
<td></td>
<td>pullout load tests</td>
</tr>
<tr>
<td>Tensile resistance of anchor</td>
<td>Permanent</td>
</tr>
<tr>
<td></td>
<td>• Yielding of the gross section</td>
</tr>
<tr>
<td></td>
<td>• Fracture of the net section</td>
</tr>
<tr>
<td>Temporary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Yielding of the gross section</td>
</tr>
<tr>
<td></td>
<td>• Fracture of the net section</td>
</tr>
<tr>
<td>Flexural capacity of vertical elements</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Permanent</td>
</tr>
<tr>
<td></td>
<td>• Temporary</td>
</tr>
</tbody>
</table>
Table 15.3.4.2-1 - Ultimate Unit Resistance of Anchors in Soil

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Compactness or SPT Resistance (Blows per 300 mm)</th>
<th>Anchor Resistance Q_a (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand and Gravel</td>
<td>Loose 4-10</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td>Medium 10-30</td>
<td>220</td>
</tr>
<tr>
<td></td>
<td>Dense 30-50</td>
<td>290</td>
</tr>
<tr>
<td>Sand</td>
<td>Loose 4-10</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Medium 10-30</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td>Dense 30-50</td>
<td>190</td>
</tr>
<tr>
<td>Sand and Silt</td>
<td>Loose 4-10</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Medium 10-30</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Dense 30-50</td>
<td>130</td>
</tr>
<tr>
<td>Soil Type</td>
<td>Unconfined Compressive Strength (MPa)</td>
<td>Anchor Resistance Q_a (N/mm)</td>
</tr>
<tr>
<td>Silt-Clay Mixtures</td>
<td>Stiff 0.10-0.24</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Hard 0.24-0.38</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 15.3.4.2-2 - Ultimate Unit Resistance of Anchors in Rock

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Ultimate Unit Anchor Resistance Q_a (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite or Basalt</td>
<td>730</td>
</tr>
<tr>
<td>Dolomitic Limestone</td>
<td>585</td>
</tr>
<tr>
<td>Soft Limestone/Sandstone</td>
<td>440</td>
</tr>
<tr>
<td>Slate and Hard Shale</td>
<td>365</td>
</tr>
<tr>
<td>Soft Shales</td>
<td>145</td>
</tr>
</tbody>
</table>
15.3.4.3 PASSIVE AND BEARING RESISTANCE

The resistance of vertical wall elements to passive and bearing failure should be evaluated using the procedures in Section S10.7 in LRFD for pile foundations.

15.3.4.4 STRUCTURAL RESISTANCE OF VERTICAL WALL ELEMENTS

Discrete vertical wall elements must be designed to resist all applicable earth and water pressure, surcharge, anchor and seismic loadings, and the vertical component of the anchor loads and other vertical loads within the tributary area between adjacent vertical wall elements. In designing these elements, fixed horizontal support can be assumed at each anchor level and at the bottom of the wall if the elements are sufficiently embedded below the base of the wall.

Unless beam on elastic foundation, finite element or other methods of soil-structure interaction analysis are used, the maximum bending moments in the vertical wall elements may be determined as follows if the proportional method is used to compute anchor forces and wall element stresses.

• The wall section above the top row of anchors is designed as a cantilever and \( M_{\text{max}} \) is determined as:

\[
M_{\text{max}} = 0.5pLx^2
\]  

(15.3.4.4-1)

where:

\( M_{\text{max}} \) = factored maximum flexural moment (N-mm)

\( p \) = average factored lateral pressure carried by vertical wall element (MPa)

\( L \) = spacing between vertical wall elements (mm)

\( x \) = vertical spacing of anchors (mm)

• Wall sections between rows are designed as simply-supported beams and \( M_{\text{max}} \) may be determined as:

\[
M_{\text{max}} = 0.125pLx^2
\]  

(15.3.4.4-2)

This moment is considered to be the positive design moment between anchors and the negative design moment at the anchors.
• If the wall is not embedded or the lateral resistance of the embedded length is neglected, the wall section below the lowest anchor level may be designed as a cantilever and $M_{\text{max}}$ may be determined using Equation 15.3.4.4-1.

• If the embedded portion of the vertical wall element is assumed to provide support and embedment is not required to resist unbalanced forces due to base instability, calculate the maximum bending moment in the section of wall below the lowest row of anchors may be determined as:

$$M_{\text{max}} = 0.333pLx^2 \quad (15.3.4.4-3)$$

• Wall sections of vertical wall elements spanning three or more equally spaced anchor levels may be designed as continuous beams and $M_{\text{max}}$ may be determined as:

$$M_{\text{max}} = 0.1pLx^2 \quad (15.3.4.4-4)$$

If the variation in lateral pressure with depth is large, moment diagrams should be constructed to provide improved accuracy.

15.3.4.5 FACING ELEMENTS

The maximum spacing between vertical wall elements should be determined based on the relative stiffness of the vertical elements and the type and condition of soil to be supported. The horizontal spacing between vertical elements typically varies between 2 to 3 m. If timber facing is specified, the timber shall be stress-grade pressure treated in conformance with Section 8 of LRFD.

Facing may be designed assuming simple support between elements, with or without soil arching, or assuming the facing behaves as a continuous support across several elements. Based on these assumptions, the value of the maximum factored flexural moment on a unit width or height of facing may be determined as follows:

• For simple spans without soil arching

$$M_{\text{max}} = 0.125pL^2 \quad (15.3.4.5-1)$$

where:

$M_{\text{max}} = \text{factored maximum flexural moment on a unit width or height of facing (N-mm)}$
p = average factored lateral pressure acting on vertical wall element (MPa)

L = spacing between vertical wall elements or other facing supports (mm)

- Simple span (soil arching)
  \[ M_{\text{max}} = 0.083pL^2 \]  
  (15.3.4.5-2)

- Continuous
  \[ M_{\text{max}} = 0.10pL^2 \]  
  (15.3.4.5-3)

If the variation in lateral pressure with depth is large, moment diagrams should be constructed to provide improved accuracy.

Equation 15.3.4.5-1 is applicable for simply supported facing behind which the soil will not arch between vertical supports (e.g., in soft cohesive soils or for rigid concrete facing placed tightly against the in-place soil). Equation 15.3.4.5-2 is applicable for simply supported facing behind which the soil will arch between vertical supports (e.g., in granular or stiff cohesive soils with flexible facing or rigid facing behind which there is sufficient space to permit the in-place soil to arch). Equation 15.3.4.5-3 is applicable for facing which is continuous over several vertical supports (e.g., reinforced shotcrete).

15.3.4.6 OVERALL STABILITY

The overall stability of anchored wall should be evaluated using the procedures described for conventional retaining walls and abutments. In performance of the slope stability analyses, consideration should be given to using a method of analysis (e.g., STABL5; Carpenter, 1986) that includes the effects of anchor forces in assessing the resistance of a slope to instability.

15.4 MECHANICALLY-STABILIZED EARTH RETAINING WALLS

15.4.1 General Design Considerations

Mechanically-stabilized earth (MSE) walls are comprised of a reinforced soil mass and a discrete modular facing which is vertical or near vertical.
The reinforced soil mass consists of select granular backfill meeting the requirements of Division II. The tensile reinforcements may be proprietary and may employ either metallic (i.e., strip- or grid-type) or polymeric (i.e., sheet-, strip-, or grid-type).

MSE walls may be used where conventional gravity, cantilever, or counterforted concrete retaining walls are considered, and they are particularly well suited where substantial total and differential settlements are anticipated. MSE walls should not be used:

- where utilities other than highway drainage would have to be constructed within the reinforced fill zone,
- with galvanized metallic reinforcements exposed to surface or groundwater contaminated by acid mine drainage or other industrial pollutants as indicated by low pH and high chlorides and sulfates, and
- where floodplain erosion or scour may undermine the reinforced fill zone, or any supporting foundation.

MSE walls may be considered for use under the following special conditions:

- When two intersecting walls form an enclosed angle of 70E or less, the affected portion of the wall shall be designed as an internally tied bin structure with at-rest earth pressure coefficients.
- Where metallic reinforcements are used in areas of anticipated stray currents within 60 m of the structure, a corrosion expert should evaluate the potential need for corrosion control requirements.

The size of the reinforced soil mass is based on consideration of:

- requirements for stability and geotechnical strength as described for conventional walls,
- requirements for structural resistance within the reinforced soil mass and the panel units, and for the development of reinforcement extending beyond assumed failure zones, and
- traditional requirements for reinforcement length of not less than 70% of the wall height, or 2.4 m.

In addition to determining the design wall section to meet the service and strength limit states criteria for applied loads, the design
of MSE walls requires consideration of factors which can affect wall performance. These factors include:

- Seismic loading (S11.9.6)
- Use in supporting a stub abutment (S11.9.7);
- Drainage control to mitigate build up of water pressure behind wall (S11.9.9)
- Scour protection and erosion control (S11.9.10)

15.4.2 Design Procedure

The design procedure for MSE walls involves the following steps:

1. Develop design foundation and wall backfill profiles, including foundation soil and rock strata layering; random backfill and reinforced backfill compositions; engineering properties of the foundation strata and backfill material(s); and groundwater level.

2. Define additional pertinent problem conditions, including wall height; ground surface geometry (i.e., slope at base and top of wall); traffic or live surcharge loads; reinforcement type and properties; and design life.

3. Estimate loads for strength and service limit states analyses and select resistance factors for conditions analyzed.

4. Identify special conditions which need to be evaluated, such as potential loss of support at base of wall due to removal of soil, scour or erosion.

5. Select an appropriate reinforcement length based on wall height.

6. Evaluate the safety against soil failure assuming the reinforced soil mass to be a rigid body (external stability). Evaluate the reinforced soil mass and foundation soil and rock strata at all applicable strength limit states for the following failure modes: overturning (eccentricity of resultant of reaction forces); sliding; and bearing (magnitude of resultant of reaction forces).

7. Evaluate the overall stability of the reinforced soil mass against deep soil failure using an acceptable limiting equilibrium method.
8. Evaluate the internal stability of the wall elements at all applicable strength limit states. Elements to be considered include the reinforcing strips or grids (to be evaluated for pullout failure and rupture failure giving consideration to loss due to corrosion), wall face elements, and connections between reinforcement and wall face elements.

9. In areas of known seismicity, investigate the effects of earthquakes for each of the external and internal failure modes at the Extreme Event I Limit State. The liberalized eccentricities of Article 15.2.4.4 apply.

10. Evaluate settlement of the reinforced soil mass at the applicable service limit states.

11. If unacceptable safety is determined against external or internal failure, alter the appropriate design parameters such as: reinforcement type, length and spacing or backfill type (i.e., backfill with greater shear resistance). If bearing pressures or settlement magnitudes are unacceptable, improvements to the foundation material should be considered.

12. Incorporate drainage measures into the design to prevent saturation of the reinforced backfill and to intercept any surface flows containing aggressive elements.

Table 15.4.2-1 summarizes the strength and service limit states that must be evaluated for the design of MSE walls.

Walls should also be investigated for extreme event limit states using a resistance factor of 1.0 unless specified otherwise.
Table 15.4.2-1 - Strength and Service Limit States for Design of MSE Walls

<table>
<thead>
<tr>
<th>Design Consideration</th>
<th>Strength Limit State</th>
<th>Service Limit State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sliding</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Bearing Resistance</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Overturning</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Overall Stability</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Rupture of Reinforcing Elements</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Pullout of Reinforcing Elements</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Resistance of Wall Face Elements</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Resistance of Wall Reinforcing Element Connections</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Settlement</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

15.4.3 Movement at the Service Limit State

15.4.3.1 ANALYSIS OF WALL DISPLACEMENTS

The displacement of MSE walls and MSE abutments must be evaluated at the service limit state for all applicable load combinations using Load Combination Service I from Table S3.4.1-1 in LRFD. Vertical wall movements can be estimated using the procedures described in Section S10 of LRFD for foundations and considering, as applicable, differential movements along the base of walls supported on footings due to variations of contact pressure.

15.4.3.2 TOLERABLE MOVEMENT CRITERIA

The tolerable settlement of MSE walls is limited by the longitudinal deformability of the facing and the ultimate purpose of the structure. Limiting tolerable differential settlement for systems with panels less than 2.8 m² in area and a maximum joint width of 19 mm are presented in Table 15.4.3.2-1. These limits are intended to prevent open spaces in the joints. Where foundation conditions indicate large differential settlements are possible over
a short horizontal distance, a vertical full-height slip joint must be used.

Table 15.4.3.2-1 - Relationship Between Joint Width and Limiting Distortion of MSE Wall Facing

<table>
<thead>
<tr>
<th>Joint Width (mm)</th>
<th>Limiting Vertical Distortion</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>1/100</td>
</tr>
<tr>
<td>13</td>
<td>1/200</td>
</tr>
<tr>
<td>6</td>
<td>1/300</td>
</tr>
</tbody>
</table>

The tolerable movement criteria for MSE abutments should follow the guidance provided in Section 16.2.3.2 for spread footing foundations.

15.4.4 Resistance at the Strength Limit State

Mechanically-stabilized earth walls must be proportioned to ensure adequate resistance at the strength limit state based on consideration of:

- Overturning resistance of the reinforced soil zone
- Sliding of the reinforced soil zone
- Resistance of soil or rock upon which the reinforced soil zone bears
- Overall stability against deep-seated failure of foundation materials below the structure
- Structural capacity of wall facing and connections
- Pullout and rupture resistance of reinforcing elements

15.4.4.1 RESISTANCE FACTORS

Resistance factors for the design of MSE walls are presented in Table 15.4.4.1-1. The resistance factors were developed based on engineering judgment and calibration with current allowable stress design procedures for MSE walls. The provisions of Sections 5, 6 and/or 8 of LRFD should be followed in evaluating the structural resistance of wall components.

When applying resistance factors for polymeric reinforcements, it should be noted that the resistance factors for tensile strength incorporate factors for the effects of creep, aging,
environmental losses and construction damage. The resistance factors for the strength limit state in the table include a multiplier of 0.33 for the effects of construction damage and 0.5 for the effects of post-construction environmental and aging strength losses. If product-specific data is available that indicates that the effects of construction damage and/or environmental deterioration and aging are less severe, higher multipliers may be used to increase the resistance factor.

15.4.4.2 SAFETY AGAINST SOIL FAILURE

Similar to requirements in 15.2.3 for conventional retaining walls and abutments, MSE walls shall be evaluated at the strength limit state for bearing and sliding resistance, the location of the resultant force at the base of the wall and overall stability.
#### Table 15.4.4.1-1 - Resistance Factors for Mechanically-Stabilized Earth Walls

<table>
<thead>
<tr>
<th>Mechanically-Stabilized Earth Walls</th>
<th>Bearing resistance</th>
<th>Section S10.5 applies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sliding</td>
<td>Section S10.5 applies</td>
<td></td>
</tr>
<tr>
<td>Tensile resistance of metallic reinforcement</td>
<td>Strip reinforcements</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>• Yielding of gross section less sacrificial area</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>• Fracture of net section less sacrificial area</td>
<td>0.75</td>
</tr>
<tr>
<td>Grid reinforcements</td>
<td>• Yield of gross section less sacrificial area</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>• Fracture of net section less sacrificial area</td>
<td>0.75</td>
</tr>
<tr>
<td>Connectors</td>
<td>• Yielding of gross section less sacrificial area</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>• Fracture of net section less sacrificial area</td>
<td>0.60</td>
</tr>
<tr>
<td>Strength limit state tensile resistance of polymeric reinforcement</td>
<td>From laboratory creep tests of 10,000 hours minimum duration</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>From wide-width tensile test-ASTM D4595 @ 5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Polyethylene @ ULT Strain</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Polypropylene</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>• Polyester</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>• Polyamine</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>• High Density Polyethylene</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td>Service limit state tensile resistance of polymeric reinforcement</td>
<td>From laboratory creep tests of 10,000 hours minimum duration</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>From limit state strength of &quot;4b&quot;</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Ultimate pullout resistance in soil | 0.90 |

For the evaluation of sliding resistance, the coefficient of friction at the base of the reinforced soil zone should be evaluated using the angle of friction, the foundation material, or a maximum value of 30\(\text{E} \) in the absence of specific strength data for the foundation materials.
For the evaluation of bearing resistance, an equivalent footing shall be assumed having a width equal to the length of the reinforcing elements and a length equal to the length of the wall. Bearing pressures shall be computed using a uniform base pressure distribution over an effective width of footing in accordance with S10.6.3.1 and S10.6.3.2 in LRFD. The effect of eccentricity and load inclination is included in the evaluation of bearing resistance using an effective width of \((B' = B - 2e_b)\) instead of the actual width.

15.4.4.3 INTERNAL STABILITY OF REINFORCEMENTS

MSE walls shall be evaluated for internal failure by slip or rupture of the reinforcements. For these evaluations, the factored horizontal force acting on the reinforcement at any level, \(P_i\), shall be:

\[
P_i = \sigma_i h_i
\]  

(15.4.4.3-1)

where:

\(h_i\) = height of reinforced soil zone contributing horizontal load to the reinforcement at Level i, determined as the vertical distance from the mid-point between Layer i and the next overlying layer to the mid-point between Layer i and the next underlying layer (mm)

\(\sigma_i\) = factored horizontal stress at Layer i, determined as described below, for inextensible and extensible reinforcements (MPa)

**Inextensible Reinforcements**

The internal stability of structures constructed with metallic strip or grid reinforcements shall be analyzed by considering that the in situ reinforced zone can be divided in two zones, the active and resistant zones. The failure surface is assumed to be bilinear as shown in Figure 15.4.4.3-1.
The factored horizontal stress, $\sigma_H$, at each reinforcement level is:

$$\sigma_H = \gamma_p \sigma_v k$$  \hspace{1cm} (15.4.4.3-2)

where:

$\gamma_p$ = load factor for earth pressure in Table S3.4.1-2 of LRFD

$k$ = horizontal pressure coefficient (dim)

$\sigma_v$ = pressure due to the resultant vertical forces at the reinforcement level determined using a uniform pressure distribution over the effective width $(B - 2e_B)$ from S10.6.3.1.5 in LRFD (MPa)

In determining $\sigma_v$, at each level of reinforcement, evaluation of equilibrium shall consider only the forces acting at that level. Structures shall be designed using $k = k_o$ at $H_1$ above the top of the leveling pad and decreasing linearly to $k = k_a$ at 6000 mm as
shown in Figure 15.4.3-1. Below a depth of 6000 mm depth, \( k = k_a \). The earth pressure coefficients \( k_a \) and \( k_o \) shall remain the same, regardless of the external loading conditions. The values of \( k_a \) and \( k_o \) shall be taken from S3.11.5.7 in LRFD.

The maximum friction angle used to determine the horizontal force within the reinforced soil mass shall be 34E, unless the specific project select backfill is tested for frictional strength by triaxial or direct shear testing. Live loads shall be positioned for extreme force effect within the zone subjected to live loads. For this loading, the provisions of S3.11.6 in LRFD must be followed.

**Extensible Reinforcements**

The internal stability for MSE structures constructed with polymeric reinforcements must be analyzed using a tie-back wedge method approach. This approach assumes that the full shear strength of the reinforced fill is mobilized and active lateral earth pressures, \( k_a \), are developed. Because it is assumed that the full shear strength of the reinforced fill is mobilized, the soil mass bounded by the failure plane moves as a rigid body resulting in the development of active lateral earth pressures. The assumed failure plane is defined by the Rankine active earth pressure zone defined by a straight line passing through the wall toe and oriented at an angle of \( 45^\circ + \phi'/2 \) from the horizontal, for both the horizontal and sloping backfill conditions. The value of \( k_a \) is determined following S3.11.5.7 in LRFD.

The tensile force at each level of reinforcement is a function of the vertical stress induced by gravity, uniform normal surcharge multiplied by \( k_o \) and active thrust. Reinforcement tensions induced by vertical or horizontal line loads, or by point loads should be added by superposition to the tensile forces induced by the reinforced wall fill soil and the retained backfill. The method of computation shall assume an unyielding rigid wall rotating about its toe.

The value of \( k_a \) in the reinforced soil mass is assumed to be independent of all external loads, except sloping fills. If testing of the site-specific select backfill is not available, the maximum friction angle used to compute the horizontal stress within the reinforced soil mass shall not exceed 34E. Where site-specific tests are performed, the soil strength shall be evaluated at residual stress levels.
15.4.4.4 PULLOUT OF REINFORCING ELEMENTS

The pullout resistance of reinforcements shall be evaluated at each level using only the effective pullout length which extends beyond the theoretical failure surface. The length of reinforcements shall meet the following criteria:

- The minimum length of the resistant zone shall be 900 mm
- The minimum total length of reinforcement shall be 2400 mm
- The reinforcement length shall be equal at all levels

For ribbed or smooth steel reinforcing strips, the nominal pullout capacity, \( P_n = P_{fs} \) in newtons, shall be determined as:

\[
P_{fs} = g f* \gamma s Z A_s \times 10^{-9}
\]  

(15.4.4.4-1)

where:

- \( g \) = gravitational constant (9.8066 m²/sec)
- \( f* \) = apparent coefficient of friction at each reinforcement level (dim)
- \( \gamma s \) = unfactored soil density (kg/m³)
- \( Z \) = depth below effective top of wall or to reinforcement (mm)
- \( A_s \) = total top and bottom surface area of reinforcement along effective pullout length, less sacrificial thickness (mm²)

In the absence of pullout data for ribbed reinforcement strips in backfill materials conforming to Division II for MSE backfills, the value of \( f* \) shall not exceed 2.0 at ground level and may be assumed to decrease linearly to a value of \( \tan \varphi_f \) at a depth of 6000 mm, where \( \varphi_f \) is the friction angle of the backfill in the reinforced zone. If the uniformity coefficient \( (D_{60}/D_{10}) \) of backfill is less than 4, but otherwise meets Division II requirements for MSE backfills, \( f* \) should not exceed 1.2.

For smooth steel reinforcing strips, \( f* \) shall be constant at all depths and determined as:

\[
f = \tan \psi \# 0.4
\]  

(15.4.4.4-2)
where:

\[ \psi = \text{soil-reinforcement angle of friction (deg)} \]

For steel grid reinforcing systems with transverse bar spacings of 150 mm or greater, the generalized relationship for the nominal pullout capacity, \( P_N = P_{fg} \) in newtons, shall be taken as:

\[ P_{fg} = g N_p \gamma_s Z n A_b \times 10^{-9} \]  
(15.4.4.4-3)

where:

\( g \) = gravitational constant (9.8066 m/sec\(^2\))

\( N_p \) = pullout based on site-specific pullout tests, or as defined by Figure 15.4.4.4-1 (dim)

\( \gamma_s \) = unfactored soil density (kg/m\(^3\))

\( Z \) = depth below effective top of wall or to reinforcement (mm)

\( n \) = number of transverse bearing members behind failure plane (dim)

\( A_b \) = surface area of transverse reinforcement in bearing, less sacrificial thickness of cross bars (mm\(^2\))

Figure 1 indicates that \( \varphi_f \geq 34E \). This is because the data for Figure 1 is derived from data based on \( \varphi_f \) of 34E or more.

Figure 15.4.4.4-1 - Pullout Factors for Inextensible Mesh and Grid Reinforcement
For grid reinforcements with transverse spacing less than 150 mm, the nominal pullout capacity shall be calculated using the following expression:

\[ P_{fg} = 2gw\gamma_s Z f_d \tan \phi \times 10^{-9} \quad (15.4.4.4-4) \]

where:

- \( g \) = gravitational constant (9.8066 m/sec\(^2\))
- \( w \) = width of mat (mm)
- \( l \) = length of mat beyond failure plane (mm)
- \( \gamma_s \) = unfactored soil density (kg/m\(^3\))
- \( Z \) = depth below effective top of wall or to reinforcement (mm)
- \( f_d \) = coefficient of resistance to direct sliding of reinforcement (dim)
- \( \phi_f \) = internal angle of friction of reinforced soil zone (dim)

The value of \( f_d \) may be assumed to vary from 0.45 for continuous sheets to 0.80 for bar mats with transverse spacing of 150 mm. Values of \( f_d \) must be determined experimentally for each grid geometry.

For polymeric reinforcements, Equation 15.4.4.4-4 is applicable where \( f_d \) is developed for a range of normal stresses in accordance with GRI-GG-5 (GRI, 1994). Experimental values of \( f_d \) may be limited by the limit state tensile load, \( T_1 \).

15.4.4.5 DESIGN LIFE

The long-term durability of steel and polymeric reinforcements must be considered in the design of MSE walls to ensure suitable performance throughout the design life of the structure.

The structural design of galvanized steel soil reinforcements and connections shall be made on the basis of a thickness, \( E_c \), defined as:

\[ E_c = E_n - E_s \quad (15.4.4.5-1) \]

where:
For structural design, sacrificial thicknesses shall be computed for each exposed surface as follows:

- Loss of galvanizing = 0.015 mm/year for the first 2 years
  = 0.004 mm/year for subsequent years

- Loss of carbon steel = 0.012 mm/year after zinc depletion

Other corrosion-resistant coatings, if specified, shall be electrostatically applied, resin-bonded epoxy type, with a minimum application thickness of 0.40 mm.

In determining the requirements for corrosion protection, the following should be considered:

- Galvanizing is preferred to use of epoxy-bonded coatings because the long-term performance of epoxy-bonded coatings buried in soil is unknown.

- If used, epoxy coatings should have a minimum thickness of 0.38 mm to 0.46 mm when placed in granular fills with sharp angular fragments.

- The sacrificial thickness should be provided in addition to the epoxy coating.

- Alloys, such as aluminum and stainless steel, should not be used for reinforcements.

The durability of polymeric reinforcements is influenced by time, temperature, mechanical damage, stress levels, microbiological attack and changes in the molecular structure by radiation or chemical exposure. The long-term stress-strain-time behavior of these materials shall be determined from the results of controlled laboratory creep tests conducted for a minimum duration of 10,000 hours for a range of load levels on samples of the finished product in accordance with ASTM D5262 (1994). These samples must be tested in the direction in which the load will be applied, and the results extrapolated to the required design life using procedures.
outlined in ASTM D2837 (1994). From this testing, the reinforcement tensile strength shall be the lesser of:

- $T_l$ - The highest load level at which the log time-creep-strain rate continues to decrease with time within the required lifetime without either brittle or ductile failure

- $T_5$ - The tension level at which total strain is not expected to exceed 5% within the design life of the structure

The effects of aging, chemical and microbiological attack, environmental stress cracking, stress relaxation, hydrolysis and variations in the manufacturing process, as well as the effects of construction damage, shall be evaluated and extrapolated to the required design life.

In determining the durability requirements for polymeric reinforcements, the following should be considered:

- In the absence of 10,000-hour creep test data, creep reduction factors may be estimated from ASTM D4595 (1994), Mitchell and Villet (1987) and Christopher and Holtz (1985).

- Short-term EPA 9090 test data is not sufficient to provide data regarding long-term strength reduction of polymeric reinforcements. For polyolefin products, oxidation strength losses can be determined using oven aging tests (Wisse, 1990).

- Environmental and aging losses for polymeric materials vary widely depending on composition of the polymer and manufacturing process. Based on hydrolysis losses alone, up to 60% of the maximum strength of polymers can be lost during a 75-year service life under poor moisture and temperature conditions. Polypropylenes are only slightly affected by hydrolysis. Hydrolysis strength losses of polyester products can be determined following (McMahon, 1959). Aging losses have not been evaluated.

- Limited testing of the effects of construction damage on polymeric reinforcements suggest minimum reduction factors to tensile strength of 20% for sand backfills, and as much as 70% if the backfills are gravels or crushed rock.

15.4.4.6 STRUCTURAL DESIGN OF FACE PANEL

Design of the concrete face panel shall be in accordance with the provisions of Section 5 of LRFD.
15.4.5 Example Problem - Mechanically Stabilized Earth (MSE) Wall

Retaining Sandy Soils

An MSE wall is proposed as follows:

During the subsurface exploration, it was determined that the foundation soils consist predominantly of sand to a depth of 10 m beneath the base of the proposed MSE wall. Bedrock underlies the sand. After correcting for depth, the average N-value for the sand is 20. The angle of internal friction is based on the SPT data.

The seasonal high groundwater table has been identified at a depth of 2 m below the bottom of the MSE wall.

Apply a live load surcharge above the 2:1 slope.

Assume the leveling pad to be constructed of precast concrete.

Inextensible ribbed strip reinforcement is 50 mm wide by 4 mm thick and has a center-to-center spacing of 700 mm. The steel reinforcing is galvanized.

Consider a design life of 100 years.
SOLUTION

Step 1: Calculate the Unfactored Loads

(A) Determine Length of Soil Reinforcement

The minimum soil reinforcement length should be taken as the greater of either 70% of the wall height as measured from the leveling pad (4.2 m) or 2400 mm (S11.9.5.1.4). For this example, try \( L = 5400 \) mm.

\[ H = 6.0 \text{ m} \]

\[ L = 5.4 \text{ m} \]

(B) Determine Vertical Earth Pressure (EV)

Total Unit Weight of reinforced backfill:

\[ 1920 \text{ kg/m}^3 \]

Weight of reinforced backfill:

\[ P_{EV1} = (6)(L)(1920)g \]
\[ = 610260 \text{ N/m of length} \]

Total Unit Weight of fill above reinforced backfill:

\[ 2100 \text{ kg/m}^3 \]

Weight of fill above reinforced backfill

\[ P_{EV2} = \left\{(1/2)(4)(2)+(2)(L-4)\right\}(2100)g \]
\[ = 140087 \text{ N/m of length} \]

(C) Determine Live Load Surcharge (LS)

The live load surcharge is applied where vehicular load is expected to act on the backfill within a distance equal to the wall height behind the wall (S3.11.6.2).

Since the live load surcharge is not applied above the reinforcing strips, only the horizontal forces will be considered.

From Table S3.11.6.2-1, an equivalent height of soil for the design vehicular loading (\( h_{eq} \)) is estimated as:

\[ h_{eq} = 760 \text{ mm} \]

Use soil density of the backfill = 2100 kg/m\(^3\)
Determine active earth pressure coefficient, $k_a$:

\[ k_a' \cos I \frac{\cos^2 I \cos^2 \phi_f}{\cos I \cos^2 I \cos^2 \phi_f} \]  

(S3.11.5.7-3)

Figure 15.4.5-2 - Earth Pressure Distribution for MSE Wall with Broken Back Backfill Surface

\[ I = \arctan\left[\frac{2}{2H}\right] = 9 \]  
\[ \cos (I) = 0.99 \]  
\[ \sin (I) = 0.16 \]  
\[ \cos \phi_f = 0.82 \]  
\[ K_a = 0.28 \]  
\[ h = H + 2 = 8.0 \text{ m} \]

Using Equation S3.11.6.2-1:

\[ \Delta p = (k)(\gamma)(g)(h_{eq})10^9 \]

\[ = 0.28 \left( 2100 \frac{Kg}{m^3} \right) \left( 9.806 \frac{m}{Sec^2} \right) (760 \text{ mm}) \times 10^6 \frac{m^3}{mm^3} \]
The live load horizontal earth pressure acting over the entire wall given by a rectangular distribution is:

\[ P_{LS} = (\Delta p)(h) \]

\[ = 0.0044 \times 8000 \times 1000 = 35 143 \text{ N/m of length} \]

(D) Determine Horizontal Earth Pressure (EH)

The basic earth pressure should be assumed to be linearly proportional to the depth of earth and, using Equation S3.11.5.7-1, taken as:

\[ P_a = 0.5 (\gamma)g(h^2)K_a \]

and

\[ \varphi = 35E \]

Use soil density of the backfill = 2100 kg/m³

\[ h = H+2 = 8.0 \text{ m} \]

\[ K_a = 0.28 \]

The horizontal earth pressure acting over the entire wall, given by a triangular pressure distribution, is:

\[ P_a = (0.5)(2100)g(h^2)(0.28) = 184 965 \text{ N/m of length} \]

\[ P_a \text{ horiz.} = P_a \cos (\theta) = 182 448 \text{ N/m of length} \]

\[ P_a \text{ vert.} = P_a \sin (\theta) = 30 408 \text{ N/m of length} \]

(E) Summary of Unfactored Loads
Table 15.4.5-1 - Vertical Loads/Moments

<table>
<thead>
<tr>
<th>Item</th>
<th>$V_{unf}$ N/m</th>
<th>Arm about 0 m</th>
<th>Moment about 0 N-m/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{EV1}$</td>
<td>610 260</td>
<td>2.7</td>
<td>1 647 703</td>
</tr>
<tr>
<td>$P_{EV2}$</td>
<td>140 087</td>
<td>3.5</td>
<td>490 853</td>
</tr>
<tr>
<td>$P_{a,vert.}$</td>
<td>30 408</td>
<td>5.4</td>
<td>164 203</td>
</tr>
<tr>
<td>Total</td>
<td>780 755</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 15.4.5-2 - Horizontal Loads/Moments

<table>
<thead>
<tr>
<th>Item</th>
<th>$H_{unf}$ N/m</th>
<th>Arm about 0 m</th>
<th>Moment about 0 N-m/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{EH} = P_{a,horiz.}$</td>
<td>182 448</td>
<td>2.7</td>
<td>486 529</td>
</tr>
<tr>
<td>$P_{LS}$</td>
<td>35 143</td>
<td>4.0</td>
<td>140 573</td>
</tr>
<tr>
<td>Total</td>
<td>217 592</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 2: Determine the Appropriate Load Factors Using Tables S3.4.1-1 and S3.4.1-2

Table 15.4.5-3 - Load Factors and Load Combinations

<table>
<thead>
<tr>
<th>GROUP</th>
<th>EV 1</th>
<th>EV 2</th>
<th>EH (Active)</th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength I-a</td>
<td>1</td>
<td>1</td>
<td>1.5</td>
<td>1.75</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>1.35</td>
<td>1.35</td>
<td>1.5</td>
<td>1.75</td>
</tr>
<tr>
<td>Strength IV</td>
<td>1.35</td>
<td>1.35</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>Service I</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Shrinkage and temperature loading is not required for MSE walls (S11.9.2).

For the loading cases identified, additional load combination groups are either redundant or have loadings which are not applicable.
Step 3: Calculate the Factored Loads and Moments

Table 15.4.5-4 - Vertical Factored Loads

<table>
<thead>
<tr>
<th>GROUP</th>
<th>$P_{EV1}$ N/m</th>
<th>$P_{EV2}$ N/m</th>
<th>$P_{EV}$ Vertical N/m</th>
<th>TOTAL Vertical N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfactored</td>
<td>610 260</td>
<td>140 087</td>
<td>30 408</td>
<td>780 755</td>
</tr>
<tr>
<td>Strength I-a</td>
<td>610 260</td>
<td>140 087</td>
<td>45 612</td>
<td>795 959</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>823 852</td>
<td>189 117</td>
<td>45 612</td>
<td>1 058 581</td>
</tr>
<tr>
<td>Strength IV</td>
<td>823 852</td>
<td>189 117</td>
<td>45 612</td>
<td>1 058 581</td>
</tr>
<tr>
<td>Service I</td>
<td>610 260</td>
<td>140 087</td>
<td>30 408</td>
<td>780 755</td>
</tr>
</tbody>
</table>

Table 15.4.5-5 - Horizontal Factored Loads

<table>
<thead>
<tr>
<th>GROUP</th>
<th>$P_{EH}$ Horizontal N/m</th>
<th>$P_{LS}$ N/m</th>
<th>TOTAL Horizontal N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfactored</td>
<td>182 448</td>
<td>35 143</td>
<td>217 592</td>
</tr>
<tr>
<td>Strength I-a</td>
<td>273 672</td>
<td>61 501</td>
<td>335 173</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>273 672</td>
<td>61 501</td>
<td>335 173</td>
</tr>
<tr>
<td>Strength IV</td>
<td>273 672</td>
<td>0</td>
<td>273 672</td>
</tr>
<tr>
<td>Service I</td>
<td>182 448</td>
<td>35 143</td>
<td>217 592</td>
</tr>
</tbody>
</table>

Table 15.4.5-6 - Factored Moments ($M_v$)

<table>
<thead>
<tr>
<th>GROUP</th>
<th>$P_{EV1}$ N-m/m</th>
<th>$P_{EV2}$ N-m/m</th>
<th>$P_{EV}$ Vertical N-m/m</th>
<th>TOTAL Vertical N-m/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfactored</td>
<td>1 647 703</td>
<td>490 853</td>
<td>164 203</td>
<td>2 302 760</td>
</tr>
<tr>
<td>Strength I-a</td>
<td>1 647 703</td>
<td>490 853</td>
<td>246 305</td>
<td>2 384 862</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>2 224 399</td>
<td>662 652</td>
<td>246 305</td>
<td>3 133 356</td>
</tr>
<tr>
<td>Strength IV</td>
<td>2 224 399</td>
<td>662 652</td>
<td>246 305</td>
<td>3 133 356</td>
</tr>
<tr>
<td>Service I</td>
<td>1 647 703</td>
<td>490 853</td>
<td>164 203</td>
<td>2 302 760</td>
</tr>
</tbody>
</table>
Table 15.4.5-7 - Factored Moments (Mh)

<table>
<thead>
<tr>
<th>GROUP</th>
<th>$P_{EH}$ Horizontal N-m/m</th>
<th>PLS N-m/m</th>
<th>TOTAL Horizontal N-m/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unfactored</td>
<td>486 529</td>
<td>140 573</td>
<td>627 102</td>
</tr>
<tr>
<td>Strength I-a</td>
<td>729 793</td>
<td>246 003</td>
<td>975 796</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>729 793</td>
<td>246 003</td>
<td>975 796</td>
</tr>
<tr>
<td>Strength IV</td>
<td>729 793</td>
<td>0</td>
<td>729 793</td>
</tr>
<tr>
<td>Service I</td>
<td>486 529</td>
<td>140 573</td>
<td>627 102</td>
</tr>
</tbody>
</table>

Step 4: Calculate the Settlement of the MSE Wall (S10.6.2.1)

Assume embankment construction behind the MSE wall has been performed previously and that any settlement from the embankment has already occurred.

The average settlement based on D'Appolonia, et al, (1968):

\[
\rho = q B \mu_o \mu_1 / M
\]

where:

\[
q = V_{TOT}/L = 780 755/[(B)(10^{-6})] = 0.1446 \text{ MPa}
\]

\[
B = 5.4 \text{ m}
\]

\[
M = 350 \text{ tsf} = 33.5 \text{ MPa}
\]

M can be correlated with average N-value (20) from SPT data, D'Appolonia, et al, (1968)

for D/B = 0.19 \hspace{1cm} \mu_o = 0.9

for H/B = 1.9 and L/B >10 \hspace{1cm} \mu_1 = 0.7

\[
\rho = 0.015 \text{ m} = 15 \text{ mm}
\]

Step 5: Check Overturning (Eccentricity)
### Table 15.4.5-8 - Summary for Eccentricity Check

<table>
<thead>
<tr>
<th>Group/Item</th>
<th>( V_{\text{TOT}} ) N/m</th>
<th>( M_{v} ) N-m/m</th>
<th>( M_{h} ) N-m/m</th>
<th>( X_{o} ) m</th>
<th>( e ) m</th>
<th>( q_{\text{uniform}} ) MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength I-a</td>
<td>795 959</td>
<td>2 384 862</td>
<td>975 796</td>
<td>1.77</td>
<td>0.93</td>
<td>0.2248</td>
</tr>
<tr>
<td>Strength I-b</td>
<td>1 058 581</td>
<td>3 133 356</td>
<td>975 796</td>
<td>2.04</td>
<td>0.66</td>
<td>0.2597</td>
</tr>
<tr>
<td>Strength IV</td>
<td>1 058 581</td>
<td>3 133 356</td>
<td>729 793</td>
<td>2.27</td>
<td>0.43</td>
<td>0.2331</td>
</tr>
<tr>
<td>Service I</td>
<td>780 755</td>
<td>2 302 760</td>
<td>627 102</td>
<td>2.15</td>
<td>0.55</td>
<td>0.1819</td>
</tr>
</tbody>
</table>

where:

\[
X_{o} = \text{location of the resultant} = \frac{(M_{v} - M_{h})}{V_{\text{TOT}}}
\]

\[
e = \text{eccentricity} = \frac{B}{2} - X_{o}
\]

**S11.6.3.3** - Require that the location of the resultant shall be in the middle half of the base.

\[
e_{\text{max}} = \frac{B}{4} = 1.35 \text{ m}
\]

For all cases, \( e \) is less than \( e_{\text{max}} \), therefore, the design is adequate with regards to Overturning.

Since \( e \) is less than \( B/6 = 0.9 \text{ m} \), for groups Strength I-b through Strength IV, the base pressure will be trapezoidal. Since \( e \) is greater than \( B/6 \) for group Strength I-a, the base pressure distribution will be triangular. A uniform distribution will be used for both cases.

The uniform pressure is taken as the average pressure of an assumed rectangular distribution.

\[
q = \frac{V_{\text{TOT}}}{(B-2e)} = \frac{V_{\text{TOT}}}{2X_{o}}
\]

**Step 6: Check Sliding**

The factored resistance against failure by sliding from S11.9.4.1 is taken as:

\[
Q_{R} = \phi_{t} Q_{t} + \phi_{ep} Q_{ep}
\]

Because of the potential for loss of soil in front of the wall, assume

\[
\phi_{ep} Q_{ep} = 0
\]

The resistance factor for sliding resistance from Table S10.5.4-1 is:
φ_t = 0.9

Nominal shear resistance is taken as:

\[ Q_t = V \tan \delta \]

\[ \tan \delta = 0.8 \tan \phi_t = 0.8 \tan 38^\circ = 0.63 \]

Using the smallest factored total vertical force from Table 15.4.5-4, Strength I-a:

\[ V = 795,959 \text{ N/m} \]
\[ Q_R = 451,309 \text{ N/m} \]

Compare \( Q_t \) to the corresponding horizontal load for Strength I-a from Table 15.4.5-5 which is 335,173 N/m. Therefore, there is adequate protection against sliding.

Step 7: Check Bearing Resistance (S11.9.4.2)

For calculation of bearing resistance, per running meter of MSE wall, use Equation S10.6.3.1.1-1 and width (B) equal to the length of reinforcement (L) = 5.4 m.

\[ q_R = \varphi q_{ult} \]

\[ \varphi = 0.35 \text{ from Table S10.5.4-1} \]

Using Equation S10.6.3.1.2C-1:

\[ q_{ult} = (0.5) \gamma (B)(C_{w1})(N_{qm})(1E-6) + \gamma (C_{w2})(D_f)(N_{qm})(1E-6) \]

where:

\[ \gamma = 2100 \text{ kg/m}^3 \]

Using Table S10.6.3.1.2c-1 and interpolating for \( C_{w1} \) for a depth of 3m below the ground surface:

\[ D_f = 1.0 \]
\[ 1.5B+D_f = 9.1 \]
\[ C_{w1} = 0.62 \]
\[ C_{w2} = 1.0 \]

Using Equation 14.2.4.2-6:
\[ N_{ym} = N_y \cdot s_y \cdot c_y \cdot i_y \cdot d_y \]

From Table 14.2.4.2-1:

\[ N_y = 78 \]

From Table 14.2.4.2-3 and assuming \( L/B = 10 \):

\[ s_y = 0.96 \]

Using Table 14.2.4.2-5 for strip footings and assuming \( q = (1)(2100) \ g = 0.021 \) MPa:

\[ c_y = 0.70 \]

For strip footings from Table 14.2.4.2-6 with \( H/V = 0.28 \) (using unfactored values for \( H/V \)):

\[ i_y = 0.37 \]

From Barker, et al, (1991), Page 22, \( d_y = 1.0 \):

\[ N_{ym} = 78 \times (0.96) \times (0.7) \times (0.37) \times (1.0) = 19.4 \]

Using Equation 14.2.4.2-7:

\[ N_{qm} = N_q \cdot s_q \cdot c_q \cdot i_q \cdot d_q \]

From Table 14.2.4.2-1:

\[ N_q = 49 \]

From Table 14.2.4.2-2 and assuming \( L/B = 10 \):

\[ s_q = 1.08 \]

Using Table 14.2.4.2-5 - for strip footings and assuming \( q = (1)(2100) \ g = 0.021 \) MPa:

\[ c_q = 0.70 \]

For \( H/V = 0.28 \) and Table 14.2.4.2-6 for strip footings and using unfactored values for \( H/V \):

\[ i_q = 0.52 \]

From Table 4.7 Barker, et al, (1991)

\[ d_q = 1.0 \]
N_{qm} = 49 \times 1.08 \times 0.70 \times 0.52 \times 1.0 = 19.3

q_{ult} = 1.07 \text{ MPa}

q_{k} = 0.37 \text{ MPa}

A uniform base pressure distribution over an effective footing width shall be used for comparison to the bearing resistance - See Table 15.4.5-8.

Comparing \( q_{R} \) to \( q_{\text{uniform}} \) from Table 15.4.5-8. Adequate bearing capacity is available.

**Step 8: Overall Stability**

Verify overall stability against a deep-seated soil failure using a limit equilibrium method of analyses.

**Step 9: Internal Stability**

**(A) Calculate the Factored Horizontal Force Acting on the Reinforcement**

Consider the slip or the rupture of the reinforcement strips

By Equation S11.9.5.2.1-1, the factored horizontal force acting on any single layer of reinforcement shall be:

\[ P_i = \sigma_H h_i \]

where:

- \( h_i \) = height of reinforced soil zone at Level i
- \( \sigma_H \) = factored horizontal stress at Layer i = \( \gamma_p \sigma_v k \)
- \( \gamma_p \) = the load factor for vertical earth pressure or earth surcharge applied to the unfactored \( \sigma_v \). The load factor is typically applied to \( V_{TOT} \) used to calculate \( \sigma_v \) and the moments used to calculate \( X_o \) (and \( e \)).
- \( \sigma_v \) = pressure due to resultant of factored vertical forces at Level i, based on overall stability of the wall section above Level i. The value of \( \sigma_v \) is determined using a uniform pressure distribution over an effective width of \( L-2e = V_{TOT}/(B-2e) = V_{TOT}/2X_o \), for effective width see S10.6.3.1.5.
- \( k \) = lateral earth pressure coefficient which varies from \( k_o \) to \( k_a \)
Use geometry shown in general Figure S11.9.5.2.2-1, reproduced below, and problem-specific Figure 15.4.5-3

![Diagram showing determination of failure plane and earth pressure coefficients MSE wall with inextensible reinforcements.](image)

Figure S11.9.5.2.2-1 - Determination of Failure Plane and Earth Pressure Coefficients MSE Wall with Inextensible Reinforcements

Using Equation S3.11.5.7-5:

\[
kh = 1 - \sin(35) = 0.43
\]

This value applied at 1.1 m above the top of the wall.

Using Equation S3.11.5.7-4:

\[
k_a = \tan^2\left(\frac{45-35}{2}\right) = 0.27
\]

This value applies at 4.9 m below the top of the wall and lower.
Figure 15.4.5-3 - Failure Plane

Place strips at a vertical spacing of 750 mm beginning at 375 mm below the top of the wall.

Live load surcharge is not included in the calculation of the internal stability.

Based on Table 15.4.5-8, the load case which produces the smallest eccentricity and the corresponding maximum uniform pressure when live load surcharge is neglected is Strength IV (EV=1.35 and EH=1.5).

Table 15.4.5-9 - Calculation of Vertical Load at the Level of Reinforcement per M of Wall

<table>
<thead>
<tr>
<th>Layer</th>
<th>z (mm)</th>
<th>$V_{TOT}$ (N/m)</th>
<th>$M_v$ (N-m/m)</th>
<th>$M_h$ (N-m/m)</th>
<th>e (m)</th>
<th>$\sigma_v$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>375</td>
<td>244 044</td>
<td>820 229</td>
<td>19 081</td>
<td>0.583</td>
<td>0.0576</td>
</tr>
<tr>
<td>2</td>
<td>1125</td>
<td>349 538</td>
<td>1 111 847</td>
<td>43 467</td>
<td>0.357</td>
<td>0.0746</td>
</tr>
<tr>
<td>3</td>
<td>1875</td>
<td>455 717</td>
<td>1 407 164</td>
<td>82 875</td>
<td>0.206</td>
<td>0.0914</td>
</tr>
<tr>
<td>4</td>
<td>2625</td>
<td>562 581</td>
<td>1 706 182</td>
<td>140 910</td>
<td>0.082</td>
<td>0.1075</td>
</tr>
<tr>
<td>5</td>
<td>3375</td>
<td>670 131</td>
<td>2 008 900</td>
<td>221 178</td>
<td>0.032</td>
<td>0.1256</td>
</tr>
<tr>
<td>6</td>
<td>4125</td>
<td>778 366</td>
<td>2 315 319</td>
<td>327 284</td>
<td>0.146</td>
<td>0.1524</td>
</tr>
<tr>
<td>7</td>
<td>4875</td>
<td>887 286</td>
<td>2 625 437</td>
<td>462 833</td>
<td>0.263</td>
<td>0.1820</td>
</tr>
<tr>
<td>8</td>
<td>5625</td>
<td>996 891</td>
<td>2 939 256</td>
<td>631 431</td>
<td>0.385</td>
<td>0.2153</td>
</tr>
</tbody>
</table>
• $V_{TOT}$ is calculated based on the weight of the soil overlying the strip in question and includes the load factor of 1.35 and 1.50 as appropriate.

• $M_h$ is calculated based on the horizontal earth pressures with a load factor of 1.5.

• $e = B/2X_o = B/2- (M_v-M_h)/V_{TOT}$, note: Eccentricity is taken as a positive value on either side of the mid-point of the strip.

• $\sigma_v = V_{TOT}/(B-2e)$

Sample Calculations for Layers 1 and 2

$$V_{TOT} = V_1 + V_2 + V_3$$

$V_1$ = weight of soil overlying strip in sloped portion above wall (factored)

$V_2$ = weight of soil overlying strip below top of wall (factored)

$V_3$ = vertical component of factored active earth pressure

Layer 1

$V_1 = \gamma_p W_1$

$= 1.35 \times (2100 \text{ kg/m}^3) \times (9.81 \text{ m/s}^2) \times [\frac{1}{2} \times (4000 \text{ mm}) \times (2000 \text{ m}) + (5400 \text{ mm-}4000 \text{ mm})(2000 \text{ mm})]\times (10^{-6})$

$= 111 245 \text{ N/m} + 77 872 \text{ N/m} = 189 117 \text{ N/m}$

$V_2 = \gamma_p W_2$
\[ V_3 = \gamma_p \cdot p \cdot \sin i \]
\[ = 1.35 \sin (9E) \cdot [0.5 \cdot (2100 \text{ kg/m}^3) \cdot (9.81 \text{ m/s}^2) \cdot (2375 \text{ mm})^2 \cdot (0.28)] \cdot (10^{-6}) \]
\[ = 3436 \text{ N/m} \]

*Note: \( \gamma_p = 1.5 \) for earth surcharge (ES)

At Layer 1, \( V_{TOT} = 189 \, 117 \text{ N/m} + 51 \, 491 \text{ N/m} + 3436 \text{ N/m} \)
\[ = 244 \, 044 \text{ N/m} \]

**Moment About Front Face of Wall at Connection to Strip**

\[ M_{v1} = (111 \, 245 \text{ N/m}) \cdot (4 \text{ m}) \cdot (2/3) \]
\[ + (77 \, 872 \text{ N/m}) \cdot (4 \text{ m} + 1/2 \cdot (5.4 \text{ m} - 4 \text{ m})) \]
\[ + (51 \, 491 \text{ N/m}) \cdot (5.4 \text{ m}/2) + (3436 \text{ N/m}) \cdot (5.4 \text{ m}) \]
\[ M_{v1} = 296 \, 653 + 365 \, 998 + 139 \, 026 + 513 + 5994 \]
\[ = 808 \, 184 \text{ N@m/m} \]

\[ M_{h1} = 1.5 \cdot \cos (9E) \cdot (2100 \text{ kg/m}^3) \cdot (9.81 \text{ m/s}^2) \cdot (0.28) \]
\[ [(0.5 \cdot (2.375))^2 \cdot (2.375)/3] \]
\[ = 19 \, 081 \text{ N@m/m} \]

**Uniform Pressure on Effective Base Width** - (For determination of pullout force and connection design force)

\[ e = \text{ABS} [5.4 \text{ m}/2 - (820 \, 231 - 19 \, 081)/244 \, 044] = 0.583 \text{ m} \]

\[ \sigma_v = \frac{244 \, 044 \text{ N/m}}{(5.4 \text{ m}^2 \cdot (0.583)) \cdot (10^{6} \text{ cm}^2)} \]
\[ = 0.0576 \text{ MPa} \]

**Layer 2**

\[ V_1 = 189 \, 117 \text{ N/m} - \text{Same as Layer 1} \]
\[V_2 = 1.35 \times (1920 \text{ kg/m}^3) \times (9.81 \text{ m/s}^2) \times (1125 \text{ mm}) \times (5400 \text{ mm}) \times (10^{-6}) = 154472 \text{ N/m}\]

\[V_3 = 1.35 \times \sin(90 \times (2100 \text{ kg/m}^3) \times (9.81 \text{ m/s}^2) \times (0.28) \times [0.5 \times (3125 \text{ mm})^2] \times (10^{-6}) = 5949 \text{ N/m}\]

\[V_{TOT} = 349537 \text{ N/m}\]

\[M_{v2} = (111245 \text{ N/m}) \times (4\text{m}) \times (2/3) + (77872 \text{ N/m}) \times (4\text{m} + 1/2 \times (5.4\text{ m} - 4\text{ m})) + (154472 \text{ N/m}) \times (5.4\text{m}/2) + (5949 \text{ N/m}) \times (5.4\text{ m}) = 1\ 111\ 844 \text{ N/m}\]

\[M_{h2} = 1.5 \times \cos(90 \times (2100 \text{ kg/m}^3) \times (9.81 \text{ m/s}^2) \times (0.28) \times [0.5 \times (3.125)^2] \times (3.125)/3] = 43\ 467 \text{ N/m}\]

\[e = \text{ABS}(5.4\text{ m}/2 - (1\ 111\ 844 - 43\ 467)/349\ 537] = 0.357 \text{ m}\]

\[\sigma_v = \frac{349\ 537}{(5.45\ 57 \times 0.357)^{10^{-6}}} = 0.0746 \text{ MPa}\]
Table 15.4.5-10 - Summary of Horizontal Load per M of Wall

<table>
<thead>
<tr>
<th>Layer</th>
<th>z (mm)</th>
<th>K</th>
<th>$\sigma_y$ MPa</th>
<th>$\sigma_H$ MPa</th>
<th>$h_i$ mm</th>
<th>$P_i$ N/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>375</td>
<td>0.39</td>
<td>0.0576</td>
<td>0.0225</td>
<td>750</td>
<td>16.89</td>
</tr>
<tr>
<td>2</td>
<td>1125</td>
<td>0.37</td>
<td>0.0746</td>
<td>0.0276</td>
<td>750</td>
<td>20.73</td>
</tr>
<tr>
<td>3</td>
<td>1875</td>
<td>0.35</td>
<td>0.0914</td>
<td>0.0320</td>
<td>750</td>
<td>24.03</td>
</tr>
<tr>
<td>4</td>
<td>2625</td>
<td>0.33</td>
<td>0.1075</td>
<td>0.0355</td>
<td>750</td>
<td>26.65</td>
</tr>
<tr>
<td>5</td>
<td>3375</td>
<td>0.31</td>
<td>0.1256</td>
<td>0.0390</td>
<td>750</td>
<td>29.26</td>
</tr>
<tr>
<td>6</td>
<td>4125</td>
<td>0.29</td>
<td>0.1524</td>
<td>0.0443</td>
<td>750</td>
<td>33.22</td>
</tr>
<tr>
<td>7</td>
<td>4875</td>
<td>0.27</td>
<td>0.1820</td>
<td>0.0493</td>
<td>750</td>
<td>36.95</td>
</tr>
<tr>
<td>8*</td>
<td>5625</td>
<td>0.27</td>
<td>0.2153</td>
<td>0.0581</td>
<td>750</td>
<td>43.60</td>
</tr>
</tbody>
</table>

*Foundation soil assumed to partially support this layer

(B) Calculate the Corrosion Losses

Original dimensions of the reinforcing in mm is:

width = 50
thickness = 4

The thickness of the galvanizing is given as 0.086 mm/side.

For structural design, sacrificial thicknesses computed for each exposed surface as follows from S11.9.8.

Loss of galvanizing = 0.015 mm/year for first 2 years
0.004 mm/year for subsequent years

Loss of carbon steel = 0.012 mm/year after zinc depletion

Thickness after first two years (per side):

0.086 - 2(0.015) = 0.056 mm

Service life for remaining zinc:
0.056/0.004 = 14 years

Total service life for zinc = 16 years

Loss of steel:
Remaining Design life = 100 - 16 = 84 years

Loss of steel over design life (per side)

= (84) 0.012 = 1.01 mm

Cross-sectional area after 100 years:

width = 47.98 mm

thickness = 1.98 mm

(C) Check for Breakage of the Strip

With a c/c spacing of 0.70 m, there will be 1.43 strip per meter of width

Using Layer 7, the maximum loaded strip without support from the foundation soil, \( P_i = 0.0474 \times 1000 \text{ mm} \times 750 \text{ mm} = 35\,550 \text{ N/m} \)

Tension Load Per Strip = 24 885 N

Maximum Tensile Stress = Tension Load/x-section area

\[
= \frac{24\,866}{(47.98)(1.98)}
\]

= 262 MPa

Allowable Tensile Stress in the Reinforcement = \( \varphi F_u \)

where:

\( \varphi \) is the tensile resistance of metallic strip reinforcement for yielding of gross section less sacrificial area

\( \varphi = 0.90 \) \hspace{1cm} Table 11.5.6-1

\( F_u \) is the minimum tensile strength

\( F_u = 400 \text{ MPa} \)

Factored Tensile Stress = 360 MPa > 262 MPa

Since the allowable tensile stress is greater than the maximum stress per strip of reinforcement, adequate protection against breakage is provided.
(D) Design of Panel Connection

By S11.9.5.1.2, the horizontal force used to design the connection to the panels may be taken as not less than 85% of the maximum calculated force as determined above, except for the lower one-half of the structure where it shall be 100% of the maximum force calculated above.

Horizontal forces, as given in Table 15.4.5-11, should be used for design of connections.

Table 15.4.5-11 - Summary of Maximum Horizontal Forces for Connection of Strip to Facing

<table>
<thead>
<tr>
<th>Layer</th>
<th>$P_i$ N/mm</th>
<th>Force/Strip N</th>
<th>Connection Design Force/Strip N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.89</td>
<td>11 821</td>
<td>10 048</td>
</tr>
<tr>
<td>2</td>
<td>20.73</td>
<td>14 513</td>
<td>12 336</td>
</tr>
<tr>
<td>3</td>
<td>24.03</td>
<td>16 820</td>
<td>14 297</td>
</tr>
<tr>
<td>4</td>
<td>26.65</td>
<td>18 655</td>
<td>15 856</td>
</tr>
<tr>
<td>5</td>
<td>29.26</td>
<td>20 485</td>
<td>20 485</td>
</tr>
<tr>
<td>6</td>
<td>33.22</td>
<td>23 252</td>
<td>23 252</td>
</tr>
<tr>
<td>7</td>
<td>36.95</td>
<td>25 865</td>
<td>25 865</td>
</tr>
<tr>
<td>8*</td>
<td>43.60</td>
<td>30 520</td>
<td>30 520</td>
</tr>
</tbody>
</table>

*Connection at Layer 8 shall be designed for force in Layer 7 (see note Table 15.4.5-9)

Sample Calculation for Layer 1:

- $P_1 = 16.89$ N/mm
- Force/Strip = (16.89 N/mm)
  = (700 mm)
  = 11 821 N
- Connection Design Force = (0.85) (11 821 N)
  = 10 048 N

(E) Calculate the Pullout Resistance of the Reinforcement using Equation S11.9.5.3-1

$$P_{fs} = g \cdot f^* \cdot \gamma_s \cdot Z \cdot A_s \cdot 10^{-9}$$
where:

\[ f^* = \text{apparent coefficient of friction at each reinforcement level} \]

at the ground level, \( f^* = 2.0 \)

at 6 m, \( f^* = \tan \phi_f = 0.65 \)

\[ \phi_f = \text{the friction angle of the backfill within the reinforced volume} = 33^\circ \]

\[ \gamma_s = \text{soil density} = 1920 \, \text{kg/m}^3 \]

\[ Z = \text{depth below effective top of wall (} H_1 \text{ on Figure 15.4.5-3)} \]

\[ A_s = \text{total top and bottom surface area of reinforcement along the effective pullout length beyond the failure plane specified in Figure 15.4.5-3 less any sacrificial thickness.} \]

From Table S11.5.6-1:

Ultimate Pullout Resistant Factor = 0.85

Table 15.4.5-12 - Pullout Capacity

<table>
<thead>
<tr>
<th>Layer</th>
<th>Z (mm)</th>
<th>(f^*)</th>
<th>Pullout Length (mm)</th>
<th>(A_s) (mm(^2))</th>
<th>(P_{fs}) N</th>
<th>(\phi P_{fs}) N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1475</td>
<td>1.72</td>
<td>3270</td>
<td>313 815</td>
<td>14 991</td>
<td>12,742</td>
</tr>
<tr>
<td>2</td>
<td>2225</td>
<td>1.58</td>
<td>3270</td>
<td>313 815</td>
<td>20 737</td>
<td>17,626</td>
</tr>
<tr>
<td>3</td>
<td>2975</td>
<td>1.43</td>
<td>3270</td>
<td>313 815</td>
<td>25 218</td>
<td>21,435</td>
</tr>
<tr>
<td>4</td>
<td>3725</td>
<td>1.29</td>
<td>3375</td>
<td>323 892</td>
<td>29 347</td>
<td>24,945</td>
</tr>
<tr>
<td>5</td>
<td>4475</td>
<td>1.15</td>
<td>3825</td>
<td>367 078</td>
<td>35 542</td>
<td>30,211</td>
</tr>
<tr>
<td>6</td>
<td>5225</td>
<td>1.01</td>
<td>4275</td>
<td>410 263</td>
<td>40 621</td>
<td>34,528</td>
</tr>
<tr>
<td>7</td>
<td>5975</td>
<td>0.86</td>
<td>4725</td>
<td>453 449</td>
<td>44 061</td>
<td>37,452</td>
</tr>
<tr>
<td>8</td>
<td>6725</td>
<td>0.72</td>
<td>5175</td>
<td>496 634</td>
<td>45 340</td>
<td>38,539</td>
</tr>
</tbody>
</table>

\[ Z \text{ is measured down from a height } H_1 \text{ above the bottom of the wall as opposed to } z \text{ which is measured down from the top of the wall (see Figure 15.4.5-3).} \]
From S11.9.5.3:

- Pullout length must be at least 900 mm
- Minimum Total length shall be 2400 mm

Since the pullout capacity provided by the resisting zone of Figure 15.4.5-3 ($\phi_P_{fs}$ of Table 15.4.5-12) is greater than the horizontal force applied to each strip (Table 15.4.5-11), adequate protection against pullout is provided.

Sample Calculation for Layer 1:

$$Z = 7100 \text{ mm} - 6000 \text{ mm} + 375 \text{ mm} = 1475 \text{ mm} \text{ (see Figure 15.4.5-3)}$$

$$f^* = 2.0 - (2.0 - 0.65) \frac{1475}{7100} = 1.72$$

$f^*$ varies from 2.0 @ $Z = 0$ to $\tan 33^\circ = 0.65$ @ $Z = 7100 \text{ mm}$

Pullout Length $= 5400 \text{ mm} - 0.3 H_I = 3270 \text{ mm}$ (Length of Resistant Zone, see Figure 15.4.5-3)

$$A_s = 2 \times (3270 \text{ mm}) \times (47.98 \text{ mm}) = 313789 \text{ mm}^2$$

$$P_{fs} = \gamma_s f^* Z A_s \times 10^{-9}$$

$$= (9.81) (1.72) (1920) (1475) (313789) \times 10^{-9}$$

$$= 14994 \text{ N}$$

$$\phi_P_{fs} = 0.85 (14994) = 12745 \text{ N}$$
REFERENCES


McMahon, 1959


Wisse, 1990
16.1 OBJECTIVE OF THE LESSON

The objective of this lesson is to summarize the design provisions for railing systems, deck joints and bridge bearings. Design examples for both concrete and post-type railings are included to familiarize the student with the analytical design procedures required for design for vehicular collision.

16.2 OVERVIEW OF RAILING SYSTEMS

The railing section of this lecture is based on the Second Edition up to and including the 2000 Interim.

16.2.1 Traffic Railing

16.2.1.1 RAILING SYSTEMS REQUIREMENTS

Traffic railings are used on bridges constructed for the exclusive use of highways. The primary purpose of traffic railings shall be to contain and redirect vehicles using the structure. All new vehicle traffic barrier systems, traffic railings and combination railings must be shown to be structurally and geometrically crashworthy.

Consideration should be given to:

- satisfying applicable warrants
- protection of the occupants of a vehicle in collision with the railing,
- protection of other vehicles near the collision,
- protection of persons and property on roadways and other areas underneath the structure,
- possible future rail upgrading,
- railing cost effectiveness, and
- appearance and freedom of view from passing vehicles.

In high-speed rural areas, the approach end of a parapet or a railing is required to either have a crashworthy configuration or be shielded by a crashworthy traffic barrier. In addition, an approach railing system is required at the beginning of all bridge railings in high-speed rural areas. The bridge approach railing system should include a transition from the guardrail system to the rigid bridge railing system which is capable of providing lateral resistance to an errant vehicle.
The approach guardrail system shall have a crashworthy end terminal at its nosing.

Railings, their connection to the deck and the deck overhang, are to be designed to satisfy the requirements of:

- the strength limit state under all applicable load combinations, other than collision, per se, and; the requirements of the extreme event limit state under vehicular collision conditions.

16.2.1.2 PERFORMANCE LEVEL SELECTION CRITERIA

Seven test levels are specified by the Specification. It is intended that six of these levels correspond to the six test levels indicated in the update of NCHRP Report 350. These test levels are designated as Test Level 1 through 6. In addition, the LRFD Specifications has an additional test level, Test Level 5A. This test level was added to provide a separate designation to the 1060 mm high railing systems used successfully in the past. The required minimum load capacity of the test levels vary with Test Level 1 having the least load capacity. The choice of a test level is a function of the bridge alignment, traffic volume and speed. The seven test levels and the recommended site conditions for the use of each of them are listed below.

- **TL-1** - Test Level One - taken to be generally acceptable for work zones with low posted speeds and very low volume, low-speed local streets;

- **TL-2** - Test Level Two - taken to be generally acceptable for work zones and most local and collector roads with favorable site conditions as well as work zones and where a small number of heavy vehicles is expected and posted speeds are reduced;

- **TL-3** - Test Level Three - taken to be generally acceptable for a wide range of high-speed arterial highways with very low mixtures of heavy vehicles and with favorable site conditions;

- **TL-4** - Test Level Four - taken to be generally acceptable for the majority of applications on high-speed highways, freeways, expressways, and Interstate highways with a mixture of trucks and heavy vehicles;

- **TL-5A** - Test Level Five-A - taken to be generally acceptable for the same applications as TL-4 when site conditions justify a higher level of rail resistance;

- **TL-5 and TL-6** - Test Levels Five and Six - taken to be generally acceptable for applications on freeways with high-speed, high-traffic volume and a higher ratio of heavy vehicles and a highway with unfavorable site conditions.
16.2.1.3 RAILING DESIGN

Due to the difficulty of analyzing the behavior of railing systems during vehicular collision, full-scale crash-testing is required to validate all new railing systems. The crash-test criteria depends on the required performance level. Table 16.2.1.3-1 lists the test requirements for each test level.

where:

\[ B = \text{Width of Vehicle} \]

\[ G = \text{Height of center of gravity from deck} \]

\[ \theta \quad \text{Collision angle} \]

Table 16.2.1.3-1 - Bridge Railing Performance Levels and Crash-Test Criteria

<table>
<thead>
<tr>
<th>Vehicle Characteristics</th>
<th>Small Automobiles</th>
<th>Pickup Truck</th>
<th>Single-Unit Van Truck</th>
<th>Van-Type Tractor-Trailers</th>
<th>Tractor-Tanker Trailers</th>
</tr>
</thead>
<tbody>
<tr>
<td>W (N)</td>
<td>7000</td>
<td>8000</td>
<td>20000</td>
<td>80000</td>
<td>220000</td>
</tr>
<tr>
<td>B (mm)</td>
<td>1700</td>
<td>1700</td>
<td>2000</td>
<td>2300</td>
<td>2450</td>
</tr>
<tr>
<td>G (mm)</td>
<td>550</td>
<td>550</td>
<td>700</td>
<td>1250</td>
<td>1630</td>
</tr>
<tr>
<td>Crash angle, ( \theta )</td>
<td>20E</td>
<td>20E</td>
<td>25E</td>
<td>15E</td>
<td>15E</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Level</th>
<th>TEST SPEEDS (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL-1</td>
<td>50 50 50 N/A</td>
</tr>
<tr>
<td></td>
<td>N/A N/A N/A N/A</td>
</tr>
<tr>
<td>TL-2</td>
<td>70 70 70 N/A</td>
</tr>
<tr>
<td></td>
<td>N/A N/A N/A N/A</td>
</tr>
<tr>
<td>TL-3</td>
<td>100 100 100 N/A</td>
</tr>
<tr>
<td></td>
<td>N/A N/A N/A N/A</td>
</tr>
<tr>
<td>TL-4</td>
<td>100 100 100 80</td>
</tr>
<tr>
<td></td>
<td>N/A N/A N/A N/A</td>
</tr>
<tr>
<td>TL-5A</td>
<td>100 100 100 N/A</td>
</tr>
<tr>
<td></td>
<td>80 N/A N/A N/A</td>
</tr>
<tr>
<td>TL-5</td>
<td>100 100 100 N/A</td>
</tr>
<tr>
<td></td>
<td>N/A N/A 80 N/A</td>
</tr>
<tr>
<td>TL-6</td>
<td>100 100 100 N/A</td>
</tr>
<tr>
<td></td>
<td>N/A N/A N/A 80</td>
</tr>
</tbody>
</table>

Once a system is crash-tested it can be used without further analysis and/or testing, provided that the proposed installation does not have features which are not present in the tested configuration that might detract from the performance of the tested railing system.

When a minor detail is changed on, or an improvement is made to a railing system that has already been tested and approved, engineering judgment and analysis should be used when determining the need for additional crash-testing.
Due to the high cost of crash-testing, analytical procedures for the design of both concrete and metal parapets were presented in an Appendix. These procedures are intended to be used for preliminary design of test specimens. In case of slight modification of a crash-tested parapet, the analytical procedures can be used to estimate the change in parapet strength. In such a case, the modified parapets can be used without further crash-testing. The design forces to be used in the design of parapets and the distribution length of the force in each direction are given in Table 16.2.1.3-2. The given values were developed through crash-testing of numerous parapet systems.

The distribution length for collision forces varies from one performance level to another based on the test vehicle. For example, $L_t$, the contact length between the vehicle and the parapet, is equal to the diameter of a single wheel for TL-1, TL-2, TL-3 and TL-4 and the length of a tandem for TL-5A, TL-5 and TL-6.

Table 16.2.1.3-2 - Design Forces for Traffic Railings

<table>
<thead>
<tr>
<th>Design Forces and Designations</th>
<th>Railing Test Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TL-1</td>
</tr>
<tr>
<td>$F_t$ Transverse (N)</td>
<td>60 000</td>
</tr>
<tr>
<td>$F_L$ Longitudinal (N)</td>
<td>20 000</td>
</tr>
<tr>
<td>$F_v$ Vertical (N) Down</td>
<td>20 000</td>
</tr>
<tr>
<td>$L_t$ and $L_L$ (mm)</td>
<td>1220</td>
</tr>
<tr>
<td>$L_v$ (mm)</td>
<td>5500</td>
</tr>
<tr>
<td>$H_v$ (min) (mm)</td>
<td>460</td>
</tr>
<tr>
<td>Minimum $H$ Height of Rail (mm)</td>
<td>685</td>
</tr>
</tbody>
</table>

The location and distribution of collision forces is shown in Figure 16.2.1.3-1. The following notations apply:

- $R_1$, $R_2$: The resistance of the rail elements
- $R'$: Total rail resistance
- $Y'$: height of total rail resistance above riding surface
Figure 16.2.1.3-1 - Metal Bridge Railing Design Forces, Vertical Location and Horizontal Distribution Length

The following geometric requirements need to be satisfied for the railing geometry to be considered satisfactory.

• Geometric Requirements:

\[ \Sigma A/H \leq 0.25 \]

See Figure 16.2.1.3-2 for the definition of values for A and H for different types of railing systems.

• Setback requirements:

Railing geometry should give a combination above or within the shaded area in Figure 16.2.1.3-3.

• Setback/clear distance requirements:

Railing should give a combination of geometric variable which are within or below the shaded area in Figure 16.2.1.3-4. The definition of the setback, S, for each type of railing system is illustrated in the Figures 16.2.1.3-2 and 16.2.1.3-4.

The following notation applies.

\[ \Sigma A = \Sigma \text{heights of railing elements} \]

C = clear distance between rails

H = height of rail system
The goal of these requirements is to minimize the possibility of a contact between the vehicle and the posts of the post-type railing system during a collision.

Figure 16.2.1.3-2 - Typical Traffic Railings
Figure 16.2.1.3-3 - Post Setback Criteria

Bridge rails in this area have met NCHRP 230 safety evaluation guidelines.

\[ \frac{A}{H} = \text{Ratio of rail contact width to height} \]

\( S = \text{Post Setback Distance (mm)} \)
The theoretical design procedures for concrete railings is based on the yield line theory. Figures 16.2.1.3-5 and 16.2.1.3-6 show the assumed failure mechanism for collision within a parapet segment and near an expansion joint, respectively. Based on these failure mechanisms, the load capacity of the parapet decreases significantly near expansion joint locations. Hence, the number of the expansion joints in the parapet should be minimized and they should be located at expansion joints in the deck. This will also enhance performance of the deck overhang and decrease the deflections of the superstructure.
Figure 16.2.1.3-5 - Yield Line Analysis of Concrete Parapet Walls for Impact within Wall Segment
Figure 16.2.1.3-6 - Yield Line Analysis of Concrete Parapet Walls for Impact Near End of Wall Segment

The first step to determine the load capacity of a concrete parapet is to determine the moment of resistance of the cross-section. $M_c$, shown in Figure 16.2.1.3-5, is the moment of resistance about a horizontal axes parallel to the longitudinal axes of the concrete parapet. $M_c$ is calculated per unit length of the parapet and, in case of parapets with sloped face, is taken as the average along the height of the parapet. The total moment of resistance about the vertical axes of the parapet cross-section is designated $(M_b + M_w H)$. $M_w$ is the moment of resistance of a unit-height strip of the parapet about the vertical axis. If the parapet has a reinforced top, the flexural resistance of the top zone in excess of $M_w$ is designated $M_b$. Figure 16.2.1.3-7 shows the resisting moments for a parapet with vertical faces.
For a typical concrete parapet with sloped face, two failure mechanisms that are different in height may develop, depending on the dimensions and reinforcement of the parapet. Figure 16.2.1.3-8 shows the two mechanisms and the height of parapet, $H$, failure length, $L_c$, corresponding to each of the two mechanisms.

\[
(M_b + M_w H)
\]

**Failure Mechanism 1  Failure Mechanism 2**

The length of failure mechanisms can be determined as follows with the appropriate substitution of $H_1$ or $H_2$ for $H$:
• For collision within a segment:

\[
L_c \frac{L_t}{2} \sqrt{\frac{\left(\frac{L_t}{2}\right)^2}{%H\left(\frac{M_b \%H}{M_c}\right)}}
\]  (16.2.1.3-1)

• For collision near an expansion joint:

\[
L_c \frac{L_t}{2} \sqrt{\frac{\left(\frac{L_t}{2}\right)^2}{%H\left(\frac{M_b \%H}{M_c}\right)}}
\]  (16.2.1.3-2)

The load capacity of the parapet, \(R_w\), can then be calculated as:

• For collision within a segment:

\[
R_w = \left(1 - \frac{L_t}{L_c \& \frac{L_t}{2}}\right) \left[8\left(\frac{M_b \%H}{M_c \%H}\right)^2\right]
\]  (16.2.1.3-3)

• For collision near an expansion joint:

\[
R_w = \left(1 - \frac{L_t}{L_c \& \frac{L_t}{2}}\right) \left[\frac{M_c L_t^2}{H}\right]
\]  (16.2.1.3-4)

The analytical design procedure for post-type railing system is also based on an assumed failure mechanisms. Post-and-rail railing systems fail through yielding of the different elements. The load capacity can be calculated using the plastic capacity of each element and by assuming a failure mode that engage a certain number of rail spans. The failure is controlled by the mode that has the least load capacity. Figure 16.2.1.3-9 shows some of the possible failure modes involving one, two or three spans of the rail. The load capacity equations are based on the assumption that the deck overhang and post anchorage to the deck will not fail before the post.
The load capacity of the post- and rail-type railing systems, \( R \), can be calculated as follows:

- For failure modes involving an odd number of railing spans in a collision away from the end of a rail segment:

\[
R_L = \frac{16M_p (N+1)(N \% 1) P_p L}{2NL_i} \tag{16.2.1.3-5}
\]
• For failure modes involving an even number of railing spans in a collision away from the end of a rail segment:

\[ R' = \frac{16M_p \cdot \%N^2P_pL}{2NL \& L_t} \quad (16.2.1.3-6) \]

• For a collision at the end of a segment involving any number of spans:

\[ R = \frac{2M_p + 2P_p\left(\sum_{i=1}^{N} L_i\right)}{2NL - L_t} \quad (16.2.1.3-7) \]

where:

\[ N = \text{number of railing spans involved in the failure mechanism} \]
\[ M_p = \text{plastic moment of horizontal rails} \]
\[ L = \text{post spacing} \]
\[ P_p = \text{ultimate resistance of single post due to a load at } \nabla \text{ from top of deck} \]

Notice that \( M_p \) should be based on consideration of the interaction of the post moments from both the transverse and longitudinal collision loads. This will also be reflected on the calculated value of \( P_p \).

The number of posts engaged in the failure mechanism should be enough to make the load on the posts at the two ends of the failure mechanism equal or less than their yield load. For any failure mechanism, the posts at the ends of the failure mode will fail if the following condition is not satisfied:

\[ \text{Post yield load capacity} > \frac{1}{2} [R \& (N \& 1)P_p] \]
In addition to the strength requirements of the railing system, the rail should have adequate height to prevent the rollover of the vehicles. The effective height of the railing system, $H_e$, should exceed the required minimum effective height, $H_{e\text{req}}$, that is determined based on the characteristics of the rail and the type of vehicle using the following equation:

$$H_e' = \frac{G \cdot W \cdot B}{2F_t}$$

(16.2.1.3-8)

where:

- $G$ = height of vehicle center of gravity above bridge deck as specified in Table 16.2.1.3-1 (mm)
- $W$ = weight of vehicle corresponding to the required performance level, as specified in Table 16.2.1.3-1 (N)
- $B$ = out-to-out wheel spacing on an axle as specified in Table 16.2.1.3-1 (mm)
- $F_t$ = transverse force corresponding to the required performance level as specified in Table 16.2.1.3-2 (N)
Figure 16.2.1.3-11 - Traffic Railing Forces

The following conditions need to be satisfied for a parapet to be considered satisfactory:

\[ R' \geq F_t \]
\[ Y \geq H_e \]

The first condition ensures that the parapet can resist the applied collision force while the second condition ensures that vehicle rollover will not occur.

The required minimum height of concrete parapets having vertical face is 685 mm and the minimum for sloped face (shown in Figure 16.2.1.3-12) is 810 mm for Test Level 4 and 1060 mm for Test Level 5A. These heights do not precisely match those obtained from Equation 16.2.1.3-8 and were chosen based on crash-test results and on successful past experience with concrete parapets. Sloped-face test specimens that included an encroachment of 50 mm, leaving a 25 mm vertical lip at the bottom, were successfully crash-tested. Therefore, the 75 mm vertical lip does not need to be increased to account for future wearing surface.
Pedestrian railings are required along the edges of pedestrian bridges and along the outer edges of sidewalks when the highway traffic is separated from the pedestrian traffic by a traffic railing. The height and opening dimensions of pedestrian railings are required to be chosen to safeguard the pedestrians using the bridge and the public passing underneath. The minimum acceptable height of pedestrian railings is 1060 mm measured from the top of the walkway. It is common to use post-type railing as a pedestrian railing. In this case, the maximum allowed clear opening between rail elements shall be such that a 150 mm diameter sphere shall not pass through, in the lower 685 mm of the railing, and the maximum spacing in the upper portion shall be such that a 200 mm diameter sphere shall not pass through.

The elements of a post-type railing need to be designed to resist a load \( w = 0.73 \text{ N/mm} \), both transversely and vertically, acting simultaneously on each longitudinal element. In addition, a concentrated load of 890 N, which may act simultaneously with the above loads at any point and in any direction, will be applied at the top of the longitudinal element.

The posts of pedestrian railings shall be designed for a concentrated design live load applied transversely at the center of gravity of the upper longitudinal element or, for railings with a total height greater than 1500 mm, at a point 1500 mm above the top.
surface of the sidewalk. The value of the concentrated design live load for posts, $P_{LL}$, in Newtons, shall be taken as:

$$P_{LL} = 890 + 0.73 L \quad (13.8.2-1)$$

where:

$L =$ post spacing (mm)

Chain-link railing systems are usually used if there is a possibility of pedestrians or vehicles passing under the bridge. In this case, the railing system is required to protect not only the pedestrians on the bridge, but also the public underneath by being able to retain the debris that may exist on the bridge. Therefore, the chain-link mesh should be capable of retaining an average beverage bottle. Such system need to be designed to resist a pressure of $7.2 \times 10^{-4} \text{MPa}$ acting normal to the entire surface of the rail.

16.2.3 Bicycle Railings

Bicycle railings are required on bridges specifically designed to carry bicycle traffic and on bridges where specific protection of bicyclists is deemed necessary. The height of a bicycle railing is required to be not less than 1370 mm, measured from the top of the riding surface. This height is divided into a top and a bottom zones, each is 685 mm high. The rail spacing in each zone is required to satisfy the respective provisions for pedestrian railings. The need for rub-rails attached to a rail or fence to prevent snagging are controversial among many bicyclists. Where provided, rub-rails should be deep enough to protect a wide range of bicycle handlebar heights. If the rail height exceeds 1370 mm above the riding surface, design loads are to be determined by the designer. The design loads for the lower 1370 mm of the bicycle railing shall not be less than those specified for pedestrian railings, except that for railings with total height greater than 1370 mm, the design live load for posts is required to be applied at a point 1370 mm above the riding surface.

16.2.4 Combination Railings

On high-speed urban expressways where a pedestrian walkway is provided, the walkway area is required to be separated from the adjacent roadway by a combination railing. On low-speed bridges where the walkway is not separated from the roadway, a combination railing is required along the edges of the bridge. Figure 16.2.4-1 shows an example for both types of combination railings. The combination railing must conform to the geometric and load requirements of either the pedestrian or bicycle railings, whichever is applicable. The traffic railing portion of the combination railing is required to be designed for the applicable requirements of traffic railings. When designing for vehicular collision, pedestrians and bicycle design loads need not be applied simultaneously with the vehicular collision loads. Combination railings along the edges of the bridge, i.e., railings with barrier curbs, are required to be crashworthy.
with or without the sidewalk and need to be crash-tested at a speed of 70 km/hr (S13.7.1.1).

Figure 16.2.4-1 - Combination Traffic/Pedestrian Walkway

16.2.5 Curbs and Sidewalks

A sidewalk curb located on the highway traffic side of a bridge railing is considered as an integral part of the railing and needs to be included in the crash-test specimens. When curb and gutter sections with sidewalks are used on roadway approaches, the curb height for raised sidewalks on the bridge should be no more than 200 mm. If a barrier curb is required, the curb height should not be less than 150 mm. If the height of the curb on the bridge differs from that off the
bridge, it should be uniformly transitioned over a distance greater than or equal to 20 times the change in height.

16.2.6 Deck Overhang Requirements

The proper design of the deck overhangs is as important as the design of the railing itself. The deck is designed to be stronger than the railing system, such that the failure will be contained in the railing and not to extend in the deck. This makes repairs of damage due to vehicular collision easier and less expensive.

The design forces in the slab due to collision differ according to type of railing system. Three design cases need to be considered, regardless the type of rail:

• Case 1: transverse and longitudinal collision forces at the extreme event limit state
• Case 2: vertical collision force at the extreme event limit state
• Case 3: conventional design (dead load plus live load) at the strength limit state

In case of continuous concrete parapet, Case 2 produces relatively small force effects and, hence, it can be ignored.

Concrete Parapets

In case of vehicular collision, force is transmitted to the deck overhang as a tensile force (see Figure 16.2.6-1). In case of a concrete parapet, the tensile force per unit width of the deck, \( T \), is calculated as:

\[
T' = \frac{R_w}{L_c \% 2H}
\]  

(16.2.6-1)

For a deck overhang supporting a continuous concrete parapet to be satisfactory at the face of the parapet, it should have a moment of resistance, \( M_s \), in the presence of \( T \), at least equal to the sum of the moment of resistance of the parapet at its base and the dead load moments, i.e.,

\[
M_s \geq M_{DL} + M_c.
\]  

(16.2.6-2)

where:

\[
M_s = \text{Moment resistance of the overhang in the presence of the "T"}
\]

\[
M_c = \text{Moment resistance of the parapet at its base}
\]
\[ M_{DL} = \text{Dead load moment in the overhang at a section at the inside face of the parapet} \]

Even though the colliding vehicle is on the bridge, it was observed during crash-testing that the wheels near the parapet were not in contact with the deck at the moment of collision. Therefore, the required strength of the overhang does not contain a term to account for the live load moments in Case 1.

At other sections in the overhang, at some distance from the parapet, the value of \( T \) and \( M_c \) per unit length is decreased by spreading them at an angle, say 30\(^\circ\), as shown in Figure 16.2.6-2, from a length \( L_c + 2H \) at the base of the parapet for \( T \), and \( L_c \) at the base of the parapet for \( M_c \). In full-scale load tests of actual bridges with continuous concrete parapets, it was observed that the parapets distribute the loads over considerable length of the bridge. The 30\(^\circ\) angle is an arbitrary value which is thought to give a conservative value.
Figure 16.2.6-2 - Distribution of $M_c$ in the Overhang

Post-Type Railings

In case of post-type railing, the design forces for the deck overhang are calculated for each design case as follows:

Case 1: Transverse and longitudinal collision forces

The overhang moment and tensile force can be calculated as:

$$M_d' = \frac{M_{post}}{W_b \%D} \quad (16.2.6-3)$$

$$T' = \frac{P_p}{W_b \%D} \quad (16.2.6-4)$$

where:

$M_{post}$ = yield moment of the post

$M_d$ and $T$ are both force per unit width.
The factored shear load may be taken as:

$$V'_{u} = A_r F_y$$  \hspace{1cm} (16.2.6-5)

The factored resistance of deck overhangs to punching shear may be taken as:

$$V'_{r} = \varphi V_n$$  \hspace{1cm} (16.2.6-6)

$$V'_{n} = V_c \left[ W_b \% h \% 2 \left( E \% \frac{B}{2} \% \frac{h}{2} \right) \right] h$$  \hspace{1cm} (16.2.6-7)

$$V'_{c} = \left( 0.166 \% \frac{0.332}{\beta_c} \right) \sqrt{f'_c} \# \varphi 0.332 \sqrt{f'_c}$$  \hspace{1cm} (16.2.6-8)

$$\frac{B}{2} \% \frac{h}{2} \# B$$  \hspace{1cm} (16.2.6-9)

Figure 16.2.6-3 - Distribution of Moments in the Deck Overhangs Supporting Post-and-Rail Railings

The equation for the design section is:

$$2X_b + W_b = b$$
for which:

\[ \beta_c = \frac{W_b}{D} \]

where:

- \( h \) = depth of slab (mm)
- \( W_b \) = width of base plate (mm)
- \( A_f \) = area of post compression flange (mm\(^2\))
- \( F_y \) = yield strength of post compression flange (MPa)
- \( b \) = length of deck resisting post strength or shear load = \( h + W_b \)
- \( B \) = distance between centroids of tensile and compressive stress resultants in post (mm)
- \( D \) = depth of base plate (mm)
- \( E \) = edge distance from line of support to centroid of compressive stress resultant in post (mm)
- \( f'_{c} \) = 28-day compressive strength of concrete (MPa)
- \( \varphi \) = resistance factor = 1.0

The assumed distribution of forces for punching shear shall be as shown in Figure 16.2.6-4.
Figure 16.2.6-4 - Punching Shear Failure Mode

Case 2: Vertical collision force

The punching shear force and overhang moment can be calculated as:

\[ P_v' = \frac{F_v L}{L_v} \]  \hspace{1cm} (16.2.6-10)

\[ M_d' = \frac{P_v' X}{b}, b \# L \]  \hspace{1cm} (16.2.6-11)
where:

\[ L = \text{post spacing} \]

\( P_u \) and \( M_d \) are both force per unit length.

Case 3: Conventional design for dead load and live load

### 16.2.7 Design Examples

TRAFFIC PARAPET DESIGN EXAMPLES

1 - Concrete parapets

Materials:

\[ F_y = 400 \text{ MPa} \]

\[ F'c = 24 \text{ MPa} \]

Reinforcement bars:

\#10: \( A = 100 \text{ mm}^2, d = 11.3 \text{ mm} \)

\#15: \( A = 200 \text{ mm}^2, d = 16.0 \text{ mm} \)

\#20: \( A = 300 \text{ mm}^2, d = 19.5 \text{ mm} \)

Test Level - 5A

Minimum required transverse resistance = 516 000 N
(Table 16.2.1.3-2)

\[ L_t = 2440 \text{ mm} \] (Table 16.2.1.3-2)
Example 1

CONSTANT THICKNESS PARAPET

Figure 16.2.7-1 - Reinforcement of Constant Thickness Concrete Parapet

Calculate $M_c$

\[
d = 240.25 \text{ mm}
\]
\[
A_s = 300 \text{ mm}^2
\]
\[
\varphi = 1.0 \text{ for extreme event limit state (S1.3.2.1)}
\]
\[
a = \frac{A_s F_y}{0.85 f'_c b} \cdot \frac{300(400)}{0.85(24)(300)} = 19.6 \text{ mm}
\]
\[
M_c = \frac{\varphi A_s F_y}{b} \left( d \cdot \frac{a}{2} \right)
\]
\[
M_c = 92,000 \text{ N@m/mm}
\]
\[
M_b = 0 \text{ - No top beam}
\]
Calculate $M_{wH}$

$$d = 300 \& 19.5 \& \frac{11.3}{2} = 224.85 \text{ mm}$$

$$A_s = 4 \#10 = 400 \text{ mm}^2$$

$$a = 7.4 \text{ mm}$$

$$M_{wH} = \varphi F_y A_s \left[ d \& \frac{a}{2} \right] \cdot 1.0 \left[ 400 \left( 400 \frac{224.85 \& 7.4}{2} \right) \right] \cdot 35384000 \text{ Nm}$$

**COLLISION WITHIN A SEGMENT**

$L_t = 2440 \text{ mm for TL-5A}$

$$L_c = \frac{L_t \% \left( L_t \right)^2}{2} \% 8H(M_b \% M_{wH}) = 3399 \text{ mm}$$

$$R_w = \frac{8(M_b \% M_{wH})}{L_c \& \frac{L_t}{2}} % M_c \left( \frac{L_c}{L_t} \right)^2$$

$$= 130000 + 460000 = 590000 \text{ N} > 516000 \text{ N}$$

**COLLISION NEAR AN EXPANSION JOINT**

$L_c = \frac{L_t \% \left( L_t \right)^2}{2} % \frac{H(M_b \% M_{wH})}{M_c} = 2597 \text{ mm}$

$$R_w = \frac{(M_b \% M_{wH})}{L_c \& \frac{L_t}{2}} % M_c \left( \frac{L_c}{L_t} \right)^2$$

$$= 25700 + 425000 = 450700 \text{ N} < 516000 \text{ N}$$
ALTERNATIVES TO INCREASE THE LOAD CAPACITY NEAR EXPANSION JOINTS

Use sleeves and dowels to transfer a portion of the force across the joint without restraining the movement of the expansion joint, or increase the parapet reinforcement near the expansion joint. If the vertical reinforcement is increased, it may lead to heavier deck reinforcement due to the need to have the deck resistance greater than $M_c$ from the railing.

Concrete Parapets

Example 2

ALTERNATIVE PARAPET SECTION

Figure 16.2.7-2 - Reinforcement of a Typical Concrete Parapet

The moment capacity, $M_c$, at Sections I, II and III can be calculated as for the constant thickness parapet.
At Section I:

\[ a' = \frac{A_s f_y}{0.85 f_c} \left( \frac{200 \times 400}{0.85 \times 24 \times 350} \right) = 11.2 \text{ mm} \]

\[ d' = 300 + 50 + \frac{16}{2} = 242 \text{ mm} \]

\[ \phi' = 1.0 \text{ for extreme event limit state (S1.3.2.1)} \]

\[ M_{c-II} = \frac{\phi A_s f_y}{b} \left( d' \frac{a}{2} \right) = 54000 \text{ N@mm/mm} \]

Similarly, and by considering the thickness of the concrete at each section,

\[ M_{c-II} = 73500 \text{ N@mm/mm} \]

\[ M_{c-III} = 102000 \text{ N@mm/mm} \]

FAILURE MECHANISMS

\[ (M_b + M_w H) \]

Failure Mechanism 1  Failure Mechanism 2

Figure 16.2.7-3 - Possible Failure Mechanisms
Calculate $M_c$ Average

Assuming Failure Mechanism 1, $M_c$ Average =

$$\left[ \frac{54 000 \times 73 500}{2} \left( \frac{73 500 \times 92 000}{2} \times 250 \right) \right] / 1060 \times 69 400 \text{ N@m/mm}$$

Assuming Failure Mechanism 2, $M_c$ Average =

$$\left[ \frac{54 000 \times 73 500}{2} \right] \times 63 750 \text{ N@m/mm}$$

$M_b = 0$, No top beam

Calculate $M_{wH}$

Divide the section into three portions and calculate the moment capacity of each portion about the vertical axis.

Figure 16.2.7-4 - Dividing the Parapet for Calculations of $M_b + M_{wH}$
Figure 16.2.7-5 - Analysis of $M_b + M_w H$

Top portion

$A_s = 2 \#15$ for both positive and negative moment

$d_{ave} = \left(300 \% \frac{31}{2}\right) \& 75.75' \ 239.75 \ mm$

$a = \frac{A_s f_y}{0.85 f_c b} \cdot \frac{400 \times 400}{0.85 \times 24 \times 300} \ 26.14 \ mm$

$M_{positive} = M_{negative} = \phi A_s f_y \left(d \& \frac{a}{2}\right) \cdot 36 \ 270 \ 000 \ N@m$

Center portion

$A_{s, positive} = 1 \#15 = 200 \ mm^2$

$d_{positive} = \left(331 \% \frac{54}{2}\right) \& 74' \ 284 \ mm$

$a_{positive} = \frac{200 \times 400}{0.85 \times 24 \times 510} \ 7.69 \ mm$

$M_{positive} = 22 \ 410 \ 000 \ N@m$

$A_{s, negative} = 2 \#15 = 400 \ mm^2$
\[d_{\text{negative}} = 284 \text{ mm}\]

\[a_{\text{negative}} = \frac{400 \times 400}{0.85 \times 24 \times 510} \times 15.38 \text{ mm}\]

\[M_{\text{negative}} = 41 210 000 \text{ N@m}\]

\[M_{\text{ave}} = \frac{(M_{\text{positive}} + M_{\text{negative}})}{2} = 33 310 000 \text{ N@m}\]

**Bottom Portion**

\[A_{\text{positive}} = 400 \text{ mm}^2\]

\[d_{\text{positive}} = 373.5 \text{ mm}\]

\[a_{\text{positive}} = 31.37 \text{ mm}\]

\[M_{\text{positive}} = 57 250 000 \text{ N@m}\]

\[A_{\text{negative}} = 200 \text{ mm}^2\]

\[d_{\text{negative}} = 436 \text{ mm}\]

\[a_{\text{negative}} = 15.67 \text{ mm}\]

\[M_{\text{negative}} = 34 253 000 \text{ N@m}\]

\[M_{\text{ave}} = 45 751 500 \text{ N@m}\]

In calculating \(M_wH\) of the parapet, the moment resistance of the center and bottom portions are determined as follows:

The mechanism equations given in the Specification and duplicated herein are based on equal positive and negative hinge live moments. For collision within a segment, use the average of the positive and negative values. This is acceptable because the yield line mechanism for this case will have some positive moment hinges and some negative moment hinges.

For collision near expansion joint, use the moment resistance that corresponds to a moment causing tension along the inside face of parapet, i.e., \(M_{\text{positive}}\). This is required because the only yield line to form is caused by a moment causing tension along the inside face.
COLLISION WITHIN A SEGMENT

MwH:

Failure Mechanism 1:

\[ M_w H = 36270000 + 33310000 + 45751000 = 115331000 \text{ N@m} \]

Failure Mechanism 2:

\[ M_w H = 36270000 + 33310000 = 69580000 \text{ N@m} \]

**Load Capacity:**

Test Level-5A (TL-5A)

\[ L_t = 2440 \text{ mm} \]

Failure Mechanism 1:

\[ H = 1060 \text{ mm} \]

\[ L_c = \sqrt{\frac{L_t^2}{2} \cdot \frac{8H(M_b \text{ %} M_w H)}{M_c}} = 5167 \text{ mm} \]

\[ R_w' = \frac{\frac{8(M_b \text{ %} M_w H)}{L_c \frac{L_t}{2}}}{H \left( \frac{L_c \text{ %} \frac{L_t}{2}}{2} \right)^2} \cdot 676600 N > 516000 N \]

Failure Mechanism 2:

\[ H = 810 \text{ mm} \]

\[ L_c = 4142 \text{ mm} \]

\[ R_w' = 654000 N > 516000 N \]

Parapet Load Capacity = smaller of \( R_w' \text{ and } R_w^2 = 654000 N \)
COLLISION NEAR EXPANSION JOINTS

Mc: No change

M_{wH}:

Failure Mechanism 1:

\[ M_{wH} = 36\,270\,000 + 22\,410\,000 + 57\,250\,000 \]
\[ = 115\,930\,000 \text{ N@m} \]

Failure Mechanism 2:

\[ M_{wH} = 36\,270\,000 + 22\,410\,000 \]
\[ = 58\,680\,000 \text{ N@m} \]

Load Capacity:

R_{wI} = 396\,100 \text{ N} < 516\,000 \text{ N}

R_{wII} = 427\,310 \text{ N} < 516\,000 \text{ N}

Parapet capacity needs to be increased near expansion joints or lower test level may be accepted in the expansion joint region.
Example 3

RAIL AND POST PARAPET

Figure 16.2.7-6 - Geometry of Post-and-Rail Railing System

Given:

F_y = 250 MPa, f'_c = 28 MPa

Post Spacing = 1800 mm

Plastic Moment Capacity, M_p

Top Rail:

Plastic Modulus, Z = 296 000 mm^3

M_p Top Rail = f_y Z = 250 x 296 000
= 74 000 000 N@m

Other Rails:

Plastic Modulus, Z = 65 100 mm^3
\[ M_p \text{ for each rail} = 250 \times 65 \, 100 \]
\[ = 16 \, 275 \, 000 \, N\text{\@m} \]

Post:

Plastic Modulus \( Z \) = 192 000 mm\(^3\)

\[ M_p \text{ Post} = 250 \times 192 \, 000 \]
\[ = 48 \, 000 \, 000 \, N\text{\@m} \]

In calculating \( M_p \) for this example, no consideration was given to the effect of the longitudinal collision force on the plastic moment of the post.

For TL-4, \( L_t = 1070 \) mm (Table 16.2.1.3-2)

Post setback = 152 mm Figures 16.2.7-6 and 16.2.1.3-4

Maximum clear distance = 140 mm

\[ \Sigma A = 4 \times 127 = 508 \, \text{mm} \]

\[ \Sigma A/H = 508/1060 = 0.48 > 0.25 \]

From Figure 16.2.3-3 and 16.2.3-4, the chosen geometry of the parapet is acceptable.

Bottom rail probably will not yield. Therefore, this rail was ignored in the load resistance calculations of the railing system.

\[ \bar{\gamma} = \frac{\Sigma R_i Y_i}{\Sigma R_i} \]

where:

\[ R_i = \text{load resistance of the } i\text{th rail} \]

\[ Y_i = \text{height of the } i\text{th rail above the top surface of the deck} \]

\[ \Sigma R_i = \text{total resistance of the rails} = \bar{R} \]

\[ \bar{\gamma} = \text{the height of } \bar{R} \text{ above the top surface of the deck} \]

For rails made from the same material, the load resistance of the rails relative to each other is the same as the ratio between their moment capacity. Therefore, the plastic moment capacity of the rails can be used to determine \( \bar{\gamma} \) as follows:
**Lecture - 16-38**

\[ \nabla = \frac{74 \times 10^6 \times 996.5 + 16.275 \times 10^6 \times (729.5 + 462.5)}{74 \times 10^6 + 2 \times 16.275 \times 10^6} \]

\[ = 874.2 \text{ mm} \]

Required minimum effective height of rail, \( H_e' \) \[ G \& \frac{W_B}{2F_t} \]

(16.2.1.3-1)

**For TL-4**

\( G = 1250 \text{ mm (Table 16.2.1.3-1)} \)

\( W = 80000 \text{ N (Table 16.2.1.3-1)} \)

\( B = 2300 \text{ mm (Table 16.2.1.3-1)} \)

\( F_t = 240000 \text{ N (Table 16.2.1.3-2)} \)

\( H_e = 867 \text{ mm} < \nabla = 874.2 \text{ mm} \quad \text{OK} \)

**RAIL LOAD CAPACITY FOR COLLISION WITHIN A SEGMENT**

\( M_p = \Sigma M_p \text{ for top three rails} \)

\[ = 106550000 \text{ N@m} \]

\( P_p = \text{Post plastic moment/} \ \nabla \)

\[ = \frac{48000000}{874.2} = 54900 \text{ N} \]

\( L_t = 1070 \text{ mm (Table 16.2.1.3-2)} \)

See Figures 16.2.1.3-9 and 16.2.1.3-10 for various failure mechanisms.

For failure modes involving an odd number of railing spans, \( N \)

\[ R' = \frac{16M_p \% (N\&1) (N\%4) P_p L}{2NLsL_t} \]

\( N = 1 \quad R = 674000 \text{ N} \)

\( N = 3 \quad R = 256000 \text{ N} \)
For failure modes involving an even number of spans, \( N \)

\[
R' = \frac{16 M_p \%N^2 P_p L}{2NL&L_l}
\]

N = 2 \hspace{1cm} R = 343 000 N
N = 4 \hspace{1cm} R = 247 000 N
N = 6 \hspace{1cm} R = 256 000 N
N = 8 \hspace{1cm} R = 290 000 N

The load capacity of the railing system is determined by the mechanism that has the least load capacity. Other mechanisms will have no chance to form because the system would fail before the load reaches the required level to form these mechanisms. Therefore, the load capacity of the railing system = 241 600 N > 240 000 N required for TL-4 (Table 16.2.1.3-2).

Notice that for a given post cross-section and spacing, the number of rail spans involved in the failure mechanism increases as the plastic moment capacity of the rails, \( M_p \), increases. For example, if \( M_p \) was doubled, i.e., \( M_p = 213 000 \) N@m, while the post size and spacing were kept the same, the failure mechanism that would have the least load capacity would involve seven spans and would have a load capacity of 338 000 N.

Check load on posts at end of failure mode:

\# of failed posts = \# of rail spans engaged in the failure mode

\((N)-1 = 4\)

Load on failed posts = \( 4 \times P_p = 219 600 \) N

Load on the post at each end of the failure mode =

\[
\left( \frac{R & 219 600}{2} \right) \times 10 500 \times P_p < 54 900 \ N
\]

Similarly, rail needs to be checked for collisions near the end of a rail segment.

Check top rail for vertical force

\( F_v = 80 000 \) N (Table 16.2.1.3-2)
Vertical Load = \frac{80,000}{5500} \cdot 14.55 \, N/mm

Moment = \frac{WL^2}{10} \cdot 14.55 \cdot (1800)^2 = 4,714,200 \, N\cdot m

Plastic moment of top rail under vertical force = 652,500,000 \, N\cdot m > 4,714,200 \, N\cdot m

- Design the connection between the top rail and the post to resist the vertical load and the moment from the load eccentricity.

- Check the connection between the rails and the post for the effect of longitudinal collision forces.

- Design the post-to-base weld, base plate and anchor bolts to resist the plastic moment of the post.

Figure 16.2.7-7 - Distribution of Forces in the Deck Overhang

Anchors and base plate are designed to have higher resistance than the post. Post resistance is based on plastic moment, i.e., highest possible resistance. Base plate and anchor bolts resistance is based on yield strength, i.e., base plate and anchor bolts will not have significant deformations before the post collapse.

Force in anchor bolts $M_{post}/B \mu 48,000,000/205 = 234,146 \, N$

Assumed characteristics of anchor bolts:

$F_y = 400 \, MPa$

$\phi = 0.75$
Anet = 0.75 A_y

d_{bolt} = 27 \text{ mm}

Resistance of two bolts = 2 \times 0.75 \times 0.75 \times \frac{\pi (27)^2}{4} \times 400

= 257 \, 650 \text{ N}

> 234 \, 146 \text{ N}

**Base Plate**

Assume design section is at outside edge of flange (see Figure 16.2.7-7)

\( f_y = 250 \, \text{MPa} \)

Assume \( t = 35 \, \text{mm} \)

\[
M = \frac{234 \, 146 (50)}{250} \times 46 \, 829 \, N@\text{m/mm}
\]

\[
\sigma = \frac{(46 \, 829) (6)}{(35)^2} \times 229 \, \text{MPa} < f_y = 250 \, \text{MPa}
\]

**DESIGN FORCES IN DECK SLAB**

**Design Case 1:**

Transverse collision force

\[
M_d = \frac{M_{\text{post}}}{W_b \%D}, \quad \frac{48 \, 000 \, 000}{250 \%235}, \quad 98 \, 969 \, N@\text{m/mm}
\]

\[
T = \frac{P_p}{W_b \%D}, \quad \frac{54 \, 900}{250 \%235}, \quad 113 \, N/mm
\]
To have a more economical deck design, it is desirable to use smaller posts to reduce $M_d$ & $T$. However, this will result in the failure of larger number of rail spans when a vehicle collides with the railing system.

Design the deck overhang to resist $M_d$ & $T$ simultaneously using conventional reinforced concrete design procedures.

**Design Case 2:**

Calculate vertical collision force

$$P_v = \frac{F_v L_v}{L_v} = \frac{80000 \times 1800}{5500} = 26180 \text{ N}$$

$$M_d = \frac{P_v X}{b} = \frac{P_v X}{2X \% W_p}$$

For a given performance level, $M_d$ depends on the overhang length and post spacing. The variation of $M_d$ with $X$ in the given case is shown below.

Calculate punching shear (see Figure 16.2.6-4)

$$V_u = A_f f_y = 1050.6 \times 250 = 262650 \text{ N}$$

Resistance:

Assume $h = 200 \text{ mm}$

$f_c' = 28 \text{ MPa}$

$E = 80.15 \text{ mm}$

$B = 205 \text{ mm}$
Concrete punching shear strength

Resistance factor $\phi = 1.0$

$$V_c' = \left( 0.166 \% 0.332 \frac{W_b}{\beta_c} \right) \sqrt{f'_c} \# \beta 0.332 \sqrt{f'_c} \quad (16.2.6-8)$$

$$\left( 0.166 \% 0.332 \frac{W_b}{\beta_c} \right) \sqrt{f'_c} \left( 0.166 \% 0.332 \frac{W_b}{1.063} \right) \sqrt{250} = 2.53 \text{ MPa}$$

$$\phi 0.332 \sqrt{f'_c} \quad (1.0) (0.332) \sqrt{250} = 1.76 \text{ MPa}$$

$$V_c = \text{smaller of 2.53 and 1.76} = 1.76 \text{ MPa}$$

Punching shear resistance

$$V_r = \phi V_n = V_n$$

$$V_n' = V_c \left[ W_b \% h \% 2 \left( E \% \frac{B}{2} \% \frac{h}{2} \right) h \right] h \text{, assume } h = 200 \text{ mm}$$

$$= 357 400 \text{ N} > V_u$$

### 16.3 OVERVIEW OF BRIDGE JOINTS

The bridge joints provisions used in this section are based on the 1994 First Edition of the LRFD Specifications.

#### 16.3.1 General

Based on past experience, deck joints have been the source of numerous problems. Therefore, the number of movable deck joints in a structure should be minimized. Preference should be given to continuous deck systems and superstructures. Integral bridges, bridges without movable deck joints, should be considered where the length of the superstructure and flexibility of the substructures are
such that secondary stresses due to restrained movement are controlled within tolerable limits.

Where used, deck joints components are arranged to accommodate the translation and rotation of the structure at joint locations. The type of joints and surface gaps should be chosen to accommodate the movement of motorcycles, bicycles and pedestrians, as required, and should neither significantly impair the riding characteristics of the roadway, nor cause damage to vehicles. The joints should be detailed to prevent damage to the structure from water, deicing chemicals and roadway debris without significantly impairing the riding characteristics of the roadway, or causing damage to vehicles. Special details are required for joints and joint anchors for grid and timber decks, and orthotropic deck superstructures.

Longitudinal deck joints should be used on a limited basis. They should be provided only where necessary to modify the effects of differential lateral and/or vertical movement between the superstructure and substructure. Where practical, elastomeric bearings or combination bearings with the capacity for lateral movement should be used instead of longitudinal joints to accommodate differential lateral movement.

Factored force effects and factored range of movements are to be considered in the design of deck joints and their supports. Superstructure movements include those due to placement of bridge decks, volumetric changes, such as shrinkage, temperature, moisture and creep, passage of vehicular and pedestrian traffic, pressure of wind, and the action of earthquakes. Substructure movements include differential settlement of piers and abutments, tilting, flexure, and horizontal translation of wall-type abutments responding to the placement of backfill, and shifting of stub abutments due to the consolidation of embankments and in-situ soils.

Section 3 of the Specification requires that load factors of 1.0 and 1.2 be used when considering the effect of uniform temperature change on the movements at the service and strength limit states, respectively. Using the 1.2 factor is intended to ensure that the gap will not be closed under the most adverse conditions anticipated. The load factor to be used when considering the effect of temperature gradient, \( \gamma_{TG} \), is to be determined on a project-specific basis. In snow regions, force effects that may be imposed on the joints by snagging snowplow blades need to be considered in the design of joint armor, armor connections and anchors. Resistance factors and modifiers shall be taken as specified in Sections 1, 5, 6, 7 and 8, as appropriate for the joint and deck materials.

The following factors are required to be considered in determining force effects and movements:

- properties of materials in the structure, including coefficient of thermal expansion, modulus of elasticity and Poisson’s ratio,
• effects of temperature, creep and shrinkage,
• sizes of structural components,
• construction tolerances,
• method and sequence of construction,
• resistance of the joints to movements,
• substructure movements due to embankment construction,
• foundation movements associated with the consolidation and stabilization of subsoils,
• structural restraints, for which the length of superstructure affecting the movement at one of its joints shall be the length from the joint being considered to the structure's neutral point.
• skew and curvature.
• static and dynamic structural responses and their interaction, and
• approach pavement growth. Rigid approach pavements composed of cobblestone, brick, or jointed concrete will experience growth or substantial longitudinal pressure due to restrained growth.

For a curved superstructure that is laterally unrestrained by guided bearings, the direction of longitudinal movement at a bearing joint may be assumed to be parallel to the chord of the deck centerline taken from the joint to the neutral point of the structure.

16.3.2 Selection

The need of deck joints is dependant on several factors which include temperature range, possible range of foundation settlement, rigidity of the substructure, and type, material, and length of the superstructure.

Where a floorbeam design which can tolerate differential longitudinal movements resulting from relative temperature and live load response of the deck and independent supporting members, such as girders and trusses, is not practical, relief joints in the deck slab, movable joints in the stringers, and movable bearings between the stringers and floorbeams should be used. Movable joints may also be provided at abutments of single-span structures exposed to appreciable differential settlement. Intermediate deck joints should be considered for multiple-span bridges where differential settlement would result in significant overstresses.
The location and type of deck joints should be chosen to ensure proper drainage and minimize the possibility of discharging drainage on or near bridge bearing or superstructure and substructure components. The following general rules will help achieve that goal:

• Open deck joints should be avoided over roadways, railroads, sidewalks, other public areas and at the low point of sag vertical curves.

• Deck joints should be positioned with respect to abutment backwalls and wingwalls to prevent the discharge of deck drainage that accumulates in the joint recesses onto bridge seats.

• Open deck joints should be located only where drainage can be directed to bypass the bearings and be discharged directly below the joint.

• Closed or waterproof deck joints should be considered where joints are located directly above structural members and bearings that would be adversely affected by debris accumulation. Where deicing chemicals are used on bridge decks, sealed or waterproofed joints should be provided.

• For straight bridges, the longitudinal elements of deck joints, such as finger plates, curb and barrier plates, and modular joint seal support beams should be placed parallel to the longitudinal axis of the deck. For curved and skewed structures, allowance for deck end movements consistent with that provided by the bearings need to be considered.

16.3.3 Design Requirements

Deck joints are required to accommodate all ranges of possible movements including those induced during construction. These movements include probable abutment and pier movements resulting from embankment consolidation and the effect of prestress-induced shortening on the width of seals and the size of bearings. Proper construction staging may minimize these effects.

The maximum and minimum joint gap openings at the factored extreme movement are specified in Article S14.5.3.2. Specifying a minimum allowable gap is intended to provide enough room to accommodate construction tolerances without closing the gap. A wide joint gap may cause undesirable riding characteristics and, therefore, a maximum gap width is specified. If bicycle traffic is anticipated, the use of special covering floor plates in shoulder areas should be considered.

Joints in concrete decks may be armored with steel shapes, weldments or castings. Such armor is required to be recessed below roadway surfaces and be protected from snowplows.
Additional precautions to prevent damage by snowplows should be considered where the skew of the joints coincides with the skew of the plow blades, typically 30\(^\circ\) to 35\(^\circ\).

To ensure joint straightness and fit of components, the use of shapes, bars and, plates 12.5 mm or thicker, is required. To ensure appropriate fit and function, the Specification requires that following requirements be included in the contract documents:

- joint components be fully assembled in the shop for inspection and approval,
- joints and seals be shipped to the job site fully assembled, and
- assembled joints in lengths up to 18,000 mm be furnished without intermediate field splices.

During installation, deck joints need to be adjusted to account for installation temperature. In the absence of more accurate information, the installation temperature is considered as the mean shade air temperature under the structure for the 48 hours prior to joint installation in concrete structures and for the 24 hours prior to joint installation for structures where the main members are made of steel. Connections of joint supports to primary members should allow horizontal, vertical and rotational adjustments. To facilitate joint adjustment and to take full advantage of the shop preassembly of the joints, construction procedures and practices should be developed to allow joint adjustment for installation temperatures without altering the orientation of joint parts established during shop assembly.

Except for short bridges where installation temperature variations would have only a negligible effect on joint width, plans for each bearing joint should include required joint installation widths for probable installation temperatures. For concrete structures, use of a concrete thermometer and measurement of temperature in expansion joints between superstructure units may be considered.

### 16.3.4 Joint Types

Special requirements and limitations of each of the following deck joint types are presented in Article S14.5.6.

- Open joints
- Closed Joints
- Waterproofed Joints
- Joint Seals
- Poured Seals
- Compression and Cellular Seals
- Sheet and Strip Seals
- Plank Seals
- Modular Seals
16.4 OVERVIEW OF BEARINGS

The bridge bearings provisions used in this section are based on the 1994 First Edition of the LRFD Specifications.

16.4.1 Load and Movement Capabilities

The bearing chosen for a particular application is required to have appropriate load and movement capabilities. Table 16.4.1-1 and Figure 16.4.1-1 may be used as a guide. Information in Table 16.4.1-1 is based on general judgment and observation, and there will obviously be some exceptions. Bearings which are listed as suitable for limited application may work if the load and rotation requirements are not excessive.

The following notations are applicable to Table 16.4.1-1:

- **S** = Suitable
- **U** = Unsuitable
- **L** = Suitable for limited applications
- **R** = May be suitable, but requires special considerations or additional elements such as sliders or guideways
- **Long.** = Longitudinal axis
- **Trans.** = Transverse axis
- **Vert.** = Vertical axis
Table 16.4.1-1 - Bearing Suitability

<table>
<thead>
<tr>
<th>Type of Bearing</th>
<th>Movement</th>
<th>Rotation About Bridge Axis Indicated</th>
<th>Resistance to Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain Elastomeric Pad</td>
<td>L</td>
<td>L</td>
<td>S</td>
</tr>
<tr>
<td>Fiberglass Reinforced Pad</td>
<td>S</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>Cotton Duck Reinforced Pad</td>
<td>U</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>Steel-reinforced Elastomeric Bearing</td>
<td>S</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>Plane Sliding Bearing</td>
<td>S</td>
<td>S</td>
<td>U</td>
</tr>
<tr>
<td>Curved Sliding Spherical Bearing</td>
<td>R</td>
<td>R</td>
<td>S</td>
</tr>
<tr>
<td>Curved Sliding Cylindrical Bearing</td>
<td>R</td>
<td>R</td>
<td>U</td>
</tr>
<tr>
<td>Disc Bearing</td>
<td>R</td>
<td>R</td>
<td>S</td>
</tr>
<tr>
<td>Double Cylindrical Bearing</td>
<td>R</td>
<td>R</td>
<td>S</td>
</tr>
<tr>
<td>Pot Bearing</td>
<td>R</td>
<td>R</td>
<td>S</td>
</tr>
<tr>
<td>Rocker Bearing</td>
<td>S</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>Knuckle Pinned Bearing</td>
<td>U</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>Single Roller Bearing</td>
<td>S</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>Multiple Roller Bearing</td>
<td>S</td>
<td>U</td>
<td>U</td>
</tr>
</tbody>
</table>
16.4.2 Forces in the Structure Caused by Restraint of Movement

Restraint of movement results in a corresponding force or moment in the structure. Therefore, horizontal forces due to rolling friction or shear deformation of a flexible element in the bearing, and moments induced in the bridge by restraint of movement at the bearings are required to be considered in the design. These force effects should be calculated taking into account the stiffness of the bridge and the bearing. In some cases, the bearing stiffness depends on time and temperature, as well as on the magnitude of the movement.

In addition to horizontal forces, both the substructure and superstructure are required to be designed for the factored moment, $M_u$, transferred by the bearings. $M_u$ is the moment due to the tangential force caused by friction resistance at the curved surface of curved sliding bearings. This force acts about the center of the curved surface and is transmitted by the bearing. The moment imposed on individual components of the bridge structure may be different from $M_u$, depending on the location of the axis of rotation.

16.4.3 Overview of Special Design Provisions for Bearings

16.4.3.1 METAL ROCKER AND ROLLER BEARINGS

Due to the fact that cylindrical bearings contain no deformable parts and are susceptible to damage if the superstructure rotates about an axis other than the axis of the line of bearings, the rotation axis of the bearing is required to be aligned with the axis about which
the largest rotations of the supported member occur. In addition, provisions should be made to ensure that the bearing alignment does not change during the life of the bridge. In general, these bearings are unsuitable for bridges in which the axis of rotation may vary significantly under different situations, such as bridges with a large skew. They are also unsuitable for use in seismic regions because the transverse shear caused by earthquake loading can cause substantial overturning moment (SC14.7.1.1). Multiple roller bearings need to be connected by gearing to ensure that individual rollers remain parallel to each other and at their original spacing.

The allowable contact stresses in roller and rocker bearings are a function of the bearing diameter. Provisions for geometric requirements for the bearings and for the allowable contact stresses are presented in the Specification.

16.4.3.2 PTFE SLIDING SURFACES

PTFE, polytetrafluorethylene, may be used in sliding surfaces of bridge bearings to accommodate translation or rotation. PTFE may be provided in sheets or in mats woven from fibers. Unfilled sheets shall be made from PTFE resin alone, while filled sheets are made from PTFE resin uniformly blended with glass fibers, carbon fibers or other chemically-inert filler. Adding reinforcing fibers reduces creep, i.e., cold flow and wear. The filler content is required not to exceed 15% for glass fibers and 25% for carbon fibers. Sheet PTFE may contain dimples to act as reservoirs for silicone grease lubricant, effective to -35EC. The reservoirs should cover more than 20%, but less than 30% of the contact surface. These requirements are intended to provide enough lubrication while maintaining ample contact area. Dimples should not be placed to intersect the edge of the contact area so that the grease does not escape.

The friction coefficient depends on many factors, such as sliding speed, contact pressure, lubrication, temperature and properties such as the finish of the mating surface. The material properties which influence the friction coefficient are not well understood, but the crystalline structure of the PTFE is known to be important, and it is strongly affected by the quality control exercised during the manufacturing process.

To ensure uniform bearing and to allow for wear, a minimum thickness of the PTFE layer is specified. During the first few cycles of movement, small amounts of PTFE transfer to the mating surface and contribute to the very low friction achieved subsequently. This initial wear is acceptable and desirable. However, PTFE continues to wear with time and movement and is exacerbated by deteriorated or rough surfaces. Continuing wear is undesirable because it usually causes higher friction and reduces the thickness of the remaining PTFE. Unlubricated, flat PTFE wears more severely than the lubricated material. The evidence on the rate of wear is tentative. High travel speeds, such as those associated with traffic movements, appear to be more damaging than the slow ones due to thermal movements.
However, they may be avoided by placing the sliding surface on an elastomeric bearing which will absorb small longitudinal movements. No allowance for wear is made in the Specification due to the limited research available to quantify or estimate the wear as a function of time and travel. However, wear may ultimately cause the need for replacement of the PTFE, so it is wise to allow for future replacement in the original design.

Recessing is the most effective way of preventing creep in unfilled PTFE. The Specification requires that sheet PTFE which is not confined be bonded to a metal surface or to an elastomeric layer. It allows sheet PTFE confined in a recess in a rigid metal backing plate for one-half its thickness to be either bonded or unbonded. When the PTFE is bonded to the top cover layer of an elastomeric bearing, this layer should be relatively thick and hard to avoid rippling of the PTFE.

The PTFE is to be used in conjunction with a mating surface. Stainless steel is the most commonly used mating surface and is the material required by the Specification. Anodized aluminum has been sometimes used in spherical and cylindrical bearings produced in other countries and may be considered if acceptable documentation of experience is provided. The finish of the mating surface is extremely important, since it affects the coefficient of friction.Friction testing is required for the PTFE and its mating surface because of the many variables involved.

The service limit state coefficient of friction of the PTFE sliding surface is given in Table S14.7.2.5-1. However, it should be acknowledged that the value of the coefficient of friction of PTFE sliding surfaces is affected by many factors. The friction factor decreases with lubrication and increasing contact stress, but increases with sliding velocity, increasing roughness of the mating surface and decreasing temperature. Dynamic friction is smaller than static friction, and the dynamic coefficient of friction is larger for the first cycle of movement than it is for later cycles. Where friction is required to resist loads, the design coefficient of friction under dynamic loading may be taken as not more than 10% of the specified values.

The friction factors used in the 1992 AASHTO Specifications are suitable for use with dimpled, lubricated PTFE. They are too small for the flat, dry PTFE commonly used in the United States. The values presented in the LRFD Specification were modified to recognize this fact. However, coefficients of friction, somewhat smaller than those given in the Specification, are possible with care and quality control. Testing is the only reliable method for certifying the coefficient of friction and bearing behavior and, based on test results, lesser values may be used.

Contact stresses between the PTFE and the mating surface are determined at the strength limit state and are assumed to vary linearly across the contact surface. The average contact stress is to be determined by dividing the load by the projection of the contact
area onto a plane perpendicular to the direction of the load. The contact stress at the edge is to be determined based on the factored load and the extreme factored moment transferred by the bearing. The contact pressure must be limited to the allowables given in Table S14.7.2.5-1. These limits are intended to prevent excessive creep or plastic flow of the PTFE, which causes the PTFE disc to expand laterally under compressive stress and may, therefore, contribute to separation or bond failure.

16.4.3.3 BEARINGS WITH CURVED SLIDING SURFACES

Bearings with curved sliding surfaces consist of two metal parts with matching curved surfaces and a low friction sliding interface. The curved surfaces may be either cylindrical or spherical and the two surfaces must have equal nominal radii.

The geometry of a spherical bearing controls its ability to resist lateral loads, its moment rotation behavior and its frictional characteristics. The geometry is relatively easy to define, but it has some consequences which are not widely appreciated. The stress may vary over the contact surface of spherical or cylindrical bearings due to the fact that the surfaces cannot be machined as accurately as a flat smooth surface. It is important that the radius of the convex and concave surfaces be within appropriate limits. If these limits are exceeded, the bronze may crack due to hard bearing contact or there may be excessive wear and damage due to creep or cold flow of the PTFE. The stress limits used in the Specification are based on average contact stress levels.

16.4.3.4 POT BEARINGS

The basic form of pot bearings provides for rotational movements. The rotational elements of the pot bearing consist of at least a pot, a piston, an elastomeric disc and sealing rings. Pot bearings may be provided with a PTFE slider to provide for both rotation and horizontal movement. They may also be provided by a guide to limit the movements to one direction.

Softer elastomers permit rotation more readily and are preferred. Therefore, the elastomeric disc is made from a compound based on virgin natural rubber or virgin neoprene conforming to the requirements of Section 18.2 of Division II. The nominal hardness shall lie between 50 and 60 on the Shore A scale. In addition, the elastomeric disk needs to be thick enough to prevent the seal from escaping and the bearing from locking up, even under the most adverse conditions. The two factors that affect the required thickness of the disk are the design angle and the internal diameter of the pot. The minimum allowable thickness of the elastomeric disk is six times the compression of the edge of the disk due to the rotation of the piston. In calculating the design angle, the designer should account for both the anticipated movements due to loads and those due to fabrication and installation tolerances, including the rotation imposed on the bearing due to out-of-level of other bridge components, such as
undersides of prefabricated girders, and permissible misalignments during construction. Vertical deflection caused by compressive load should also be taken into account, because it will reduce the available clearance.

The allowable average stress on the elastomeric disc is largely limited by the ability of the seal to prevent escape of the elastomer. The Specification requires that the average stress on the elastomer at the service limit state not to exceed 24 MPa. This level of stress has been used as a practical upper limit for some years and most bearings have generally performed satisfactorily, but a few seal failures have occurred.

A seal is required to be used between the pot and the piston. Performance requirements under both service and strength limit states are included in the Specification. At the service limit state, seals must be adequate to prevent escape of elastomer under compressive load and simultaneously applied cyclic rotations. At the strength limit state, seals must also be adequate to prevent escape of elastomer under compressive load and simultaneously applied static rotation.

The pot and piston are required to be made from carbon steel or from stainless steel. Corrosion resistant steels, such as ASTM A709M, Grade 345W, are not recommended for applications where they may come into contact with saltwater or be permanently damp, unless their whole surface is completely corrosion protected. The yield strength and hardness of the piston is required not to exceed that of the pot. This provision is meant to avoid wear or damage on the inside surface of the pot and the consequent risk of seal failure.

Under the least favorable combination of factored displacements and rotations, the dimensions of the elements of a pot bearing are required to satisfy the following requirements:

- The pot is deep enough to permit the seal and piston rim to remain in full contact with the vertical face of the pot wall.
- Contact or binding between metal components do not prevent further displacement or rotation.

16.4.3.5 STEEL REINFORCED ELASTOMERIC BEARINGS

Steel reinforced elastomeric bearings have greater strength and superior performance compared to other elastomeric bearings. Reinforced bearings consist of alternate layers of steel reinforcement and elastomer bonded together. In addition to any internal reinforcement, bearings may have external steel plates bonded to either, or both, the upper or lower elastomer layers.

The strength and stiffness of the bearing in resisting compressive load are controlled by the thickest layer. Therefore, the Specification requires that all internal layers of elastomer be of the same thickness with the top and bottom cover layers no thicker than
70% of the internal layers. Tapered elastomer layers cause larger shear strains and bearings made with them may fail prematurely due to delamination or rupture of the reinforcement. Therefore, tapered elastomer layers are not allowed and any taper should be achieved by tapering the reinforcement layers.

The shear modulus, G, is the most important material property for design, and it is, therefore, the preferred means of specifying the elastomer. The shear modulus of the elastomer at 23EC is to be used as the basis for design. Hardness has been widely used in the past because the test for it is quick and simple. However, the results obtained from the hardness are variable and correlate only loosely with shear modulus. If the material is specified by hardness, the shear modulus to be considered in the design is taken as the least favorable value from the range of values corresponding to that hardness.Specifying the material by hardness thus imposes a slight penalty in design.

The Specification requires that shear modulus of the elastomer be between 0.55 and 1.2 MPa and a nominal hardness between 50 and 60 on the Shore A scale. Materials with a nominal hardness greater than 60 are prohibited because they generally have a smaller elongation at break, greater stiffness and greater creep than their softer counterparts. This inferior performance is generally attributed to the larger amounts of filler present. Their fatigue behavior does not differ in a clearly discernible way from that of softer materials.

Creep varies from one compound to another and is generally more prevalent in harder elastomers, but is seldom a problem if high-quality materials are used. This is particularly true because the deflection limits are based on serviceability and are likely to be controlled by live load, rather than total load. The creep values given in Table 16.4.3.5-1 are representative of neoprene and are conservative for natural rubber.

<table>
<thead>
<tr>
<th>Hardness (Shore A)</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Modulus @ 23EC</td>
<td>0.66-0.90</td>
<td>0.90-1.38</td>
<td>1.38-2.07</td>
</tr>
<tr>
<td>Creep deflection @ 25 years divided by instantaneous deflection</td>
<td>1.72</td>
<td>2.41</td>
<td>3.10</td>
</tr>
</tbody>
</table>

Shear modulus increases as the elastomer cools, but the extent of stiffening depends on the elastomer compound, time and temperature. It is, therefore, important to specify a material with low temperature properties which are appropriate for the bridge site. For
the purposes of bearing design, all bridge sites are classified as being in temperature Zones A, B, C, D or E for which design data are given in Table 16.4.3.5-2. In the absence of more precise information, Figure 16.4.3.5-1 may be used as a guide in selecting the zone required for a given region. The low temperature classification is intended to limit the force on the bridge substructure to 1.5 times the service limit state design force under extreme environmental conditions.

Figure 16.4.3.5-1 - Temperature Zones

Table 16.4.3.5-2 - Low Temperature Zones and Minimum Grades of Elastomer

<table>
<thead>
<tr>
<th>LOW TEMPERATURE ZONE</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-Year Low Temperature (EC)</td>
<td>-18</td>
<td>-30</td>
<td>-35</td>
<td>-43</td>
<td>&lt; -43</td>
</tr>
<tr>
<td>Maximum number of consecutive days when the temperature does not rise above 0°C</td>
<td>3</td>
<td>7</td>
<td>14</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Minimum low temperature elastomer grade</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Minimum low temperature elastomer grade when special force provisions are incorporated</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Steel reinforced bearings are designed to resist relatively high stresses. Their integrity depends on good quality control during manufacturing, which can only be assured by rigorous testing. Bearings designed to meet the Code provisions are required to be
tested in accordance with the requirements for steel reinforced elastomeric bearings as specified in Article 18.2.7 of Division II of the Specification.

The shape factor of a layer of an elastomeric bearing is defined as the plan area of the layer divided by the area of perimeter free to bulge. The compressive resistance of the reinforced elastomeric bearings is a function of the applied shear stress and the shape factor. Holes are strongly discouraged in steel-reinforced bearings. However, if holes are used, their effect should be accounted for when calculating the shape factor because they reduce the loaded area and increase the area free to bulge.

The relationship between the shear stress and the applied compressive load depends directly on shape factor, with higher shape factors leading to higher capacities. If movements are accommodated by shear deformations of the elastomer, they cause shear stresses in the elastomer. These add to the shear stresses caused by compressive load, so a lower load limit is allowed in case of combined shear and compressive loading.

Increases in the load to simulate the effects of impact are not required. This is because the impact stresses are likely to be only a small proportion of the total load, and also because the compressive stress limits are based on fatigue damage, the limits of which are not clearly defined.

Laminated elastomeric bearings have a nonlinear load deflection curve in compression. In the absence of information specific to the particular elastomer to be used, Figure 16.4.3.5-2 may be used as a guide.
Deflections of elastomeric bearings due to total load and to live load alone must be considered separately. Limiting instantaneous deflections is important to ensure that deck joints and seals are not damaged. A maximum relative deflection across a joint of 3.2 mm is suggested by the Specification. However, joints and seals that are sensitive to relative deflections may require tighter limits.

Long-term deflections should be considered where joints and seals between sections of the bridge rest on bearings of different design, and when estimating redistribution of forces in continuous bridges caused by settlement. Provided high-quality materials are used, the effects of creep of the bearings are unlikely to cause problems.

The maximum shear deformation of the bearing, $\Delta_s$, is required to be taken as the extreme relative displacement between the superstructure and the substructure at the bearing location, determined for the service limit state. The magnitude of the displacements is a function of the length and geometry of the superstructure, type of materials, temperature range and distribution, substructure stiffness, construction procedures and the applied loads.
This modification is required due to the fact that pier deflections sometimes accommodate a significant portion of the bridge movement, and this may reduce the movement which must be accommodated by the bearing. Construction methods may increase the bearing movement because of poor installation tolerances or poor timing of the bearing installation. If the bridge girders are lifted to allow the bearings to realign after some of the girder shortening has occurred, that may be accounted for in design. If a low friction sliding surface is installed, it will allow the movement of the superstructure relative to the substructure. In such a case, $\Delta_s$ need not be taken larger than the deformation corresponding to first slip. In any case, maximum shear deformation of the bearing must not exceed half the total elastomer thickness in order to avoid rollover at the edges and delamination due to fatigue.

Fatigue tests which formed the basis for the maximum shear deformation provision were conducted to 20,000 cycles, which represents one expansion/contraction cycle per day for approximately 55 years. The provisions will, therefore, be unconservative if the shear deformation is caused by high-cycle loading due to braking forces or vibration. The maximum shear deformation due to these high-cycle loadings should be restricted to no more than +/- 0.10 the total elastomer thickness, unless better information is available. At this strain amplitude, the experiments showed that the bearings have an essentially infinite fatigue life.

Limits on the stress in the elastomer, at the service limit state, under the effect of combined compression and rotation are presented in Article S14.7.5.3.5. These limits address two conditions. No point in the bearing is allowed to undergo net uplift, and excessive compressive stress on an edge is prevented. Uplift is required to be prevented because strain reversal in the elastomer significantly decreases its fatigue life. The interaction between compressive and rotation capacity in a bearing is illustrated in Figure 16.4.3.5-3, which is analogous to the interaction diagram for a reinforced concrete column. The following definitions apply to Figure 16.4.3.5-3:

- $n$ = number of interior layers of elastomer
- $h_{ri}$ = height of the $i_{th}$ elastomer layer (mm)
- $\sigma_s$ = stress in elastomer (MPa)
- $\Gamma$ = length of pad if rotation is about its transverse axis, or width of pad if rotation is about its longitudinal axis (mm)
- $\Theta_s$ = rotation about any axis of the pad (RAD)

Since a high shape factor is best for resisting compression, but a low one accommodates rotation most readily, the best choice represents a compromise between the two. The "balanced design" point in Figure 16.4.3.5-3, where uplift and compressive stress are...
simultaneously critical, will in many cases provide the most economical solution for a given plan geometry. Table 16.4.3.5-1 gives coordinates of the balance point for different bearing shapes.

Design rotations should be taken as the maximum sum of the effects of initial lack-of-parallelism and subsequent girder end rotation due to imposed loads and movements. A rectangular bearing should normally be oriented so that its long side is parallel to the axis about which the largest rotation occurs. The critical location in the bearing for both compression and rotation is then at the mid-point of the long side. If rotation occurs about both axes, uplift and excessive compression should be investigated in both directions.

Figure 16.4.3.5-3 Elastomeric Bearing - Interaction Between Compressive Stress and Rotation Angle
Bearings are also required to be investigated for instability at the service limit state load combinations. Equations to determine the limiting average stress for both rectangular and circular bearings are presented in Article S14.7.5.3.7. The average compressive stress given by these equations is limited to half the predicted buckling stress calculated using theoretical equations and modified to account for changes in geometry during compression. They were calibrated against experimental results. This provision will permit taller bearings and reduced shear forces compared to those permitted under previous specifications.

The reinforcement should sustain the tensile stresses induced by compression of the bearing. With the present load limitations, the minimum steel plate thickness practical for fabrication will usually provide adequate strength. Equations to calculate the minimum required reinforcement thickness are presented in Article S14.7.5.3.7. Holes in the reinforcement cause stress concentrations and, therefore, holes should be discouraged. If holes exist in the reinforcement, the minimum thickness is required to be increased by a factor equal to twice the gross width divided by the net width. This increase in steel thickness accounts for both the material removed and the stress concentrations around the hole.

16.4.3.6 ELASTOMERIC PADS

Elastomeric pads have characteristics which are different from those of steel reinforced elastomeric bearings. The Code provisions for elastomeric pads apply to the design of:

- plain elastomeric pads, PEP,
- pads reinforced with discrete layers of fiberglass, FGP, and
pads reinforced with closely spaced layers of cotton duck, CDP

Plain elastomeric pads are weaker and more flexible because they are restrained from bulging by friction alone. Slip inevitably occurs, especially under dynamic loads, causing larger compressive deflections and higher internal strains in the elastomer.

In pads reinforced with layers of fiberglass, the reinforcement inhibits the deformations found in plain pads. However, elastomers bond less well to fiberglass and the fiberglass is weaker than steel, so the fiberglass pad is unable to carry the same loads as a steel reinforced pads. Fiberglass pads have the advantage that they can be cut to size from a large sheet of vulcanized material.

Pads reinforced with closely spaced layers of cotton duck typically display high compressive stiffness and strength, obtained by the use of very thin elastomeric layers. However, the thin layers also give rise to very high rotational stiffness which could easily lead to edge loading and a higher shear stiffness than that to be found in bearings with thicker layers. This high shear stiffness leads to larger forces in the bridge, unless it is offset by the use of the PTFE slider on top of the elastomeric pad.

The elastomer needs to satisfy the same material requirements required for reinforced elastomeric pads, except that a nominal hardness between 50 and 70 on the Shore A scale is allowed. The Specification requires the shear force on the structure induced by deformation of the elastomer be based on a G value not less than that of the elastomer at 23°C. Effects of relaxation need not be considered. In general, the three types of pad, PEP, FGP and CDP behave differently, so information relevant to the particular type of pads should be used for design.

16.4.3.7 BRONZE OR COPPER ALLOY SLIDING SURFACES

Bronze or copper alloy sliding surfaces have a long history of application in the United States with relatively satisfactory performance of the different materials. However, there is virtually no research to substantiate the properties and characteristics of these bearings. Successful past experience is the best guide currently available.

Bronze or copper alloy may be used for:

- flat sliding surfaces to accommodate translational movements,
- curved sliding surfaces to accommodate translation and limited rotation, and
- pins or cylinders for shaft bushings of rocker bearings or other bearings with large rotations.
The magnitude of the horizontal forces transmitted to the substructure is a function of the vertical load and the coefficient of friction. The best available experimental evidence suggests that lubricated bronze can achieve a coefficient of friction on the order of 0.07 during its early life while the lubricant projects above the bronze surface. The coefficient of friction is likely to increase to approximately 0.10 after the surface lubrication wears away and the bronze starts to wear down into the recessed lubricant. Copper alloy or plain bronze would cause considerably higher friction. In the absence of better information, conservative coefficients of friction of 0.1 for self-lubricating bronze components and 0.4 for other types are recommended for design.

16.4.3.8 DISC BEARINGS

A disc bearing functions by deformation of a Polyether Urethane disc, which should be stiff enough to resist vertical loads without excessive deformation and yet flexible enough to accommodate the imposed rotations without lift-off or excessive stress on other components such as PTFE. Elements of a disc bearing are proportioned such that hard contact between metal components, which prevents further displacement or rotation, will not occur under the least favorable combination of design displacements and rotations at the strength limit state. Furthermore, the urethane disc should be positively located to prevent its slipping out of place.

The elastomeric disc is made from a compound based on Polyether Urethane, using only virgin materials. The appropriate material properties must be selected as an integral part of the design process, because the softest urethanes may require a limiting ring to prevent excessive compressive deflection, whereas the hardest ones may be too stiff causing an excessive resisting moment. Also, harder elastomers generally have higher ratios of creep to elastic deformation.

16.4.3.9 GUIDES AND RESTRAINTS

Guides may be used to prevent movement of a bearing in one direction. If the horizontal force becomes too large to be carried reliably and economically on a guided bearing, a separate guide system may be used. Restraints may be used to permit only limited movement in one or more directions. Guides and restraints shall have a low-friction material at their sliding contact surfaces.

16.4.3.10 OTHER BEARING SYSTEMS

The Specification allows the use of bearing systems, other than those discussed above, if approved by the bridge owner. In such case, the bearings should be carefully tested to determine whether they are adequate to resist the forces and deformations imposed on them at the service, strength and extreme event limit states without material distress and without inducing deformations detrimental to their proper functioning. The tests are required to be designed to
demonstrate any potential weakness in the system under individual compressive, shear or rotational loading, or combinations thereof. Testing under sustained and cyclic loading is also required. Two problems are often faced in planning test programs, first, the availability of bearings test equipment, and, second, the high cost of testing that may be prohibitive.